P1.T3. Financial Markets & Products

Hull, Options, Futures & Other Derivatives
Properties of Stock Options

Bionic Turtle FRM Video Tutorials

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Properties of Stock Options

- Identify the six factors that affect an option's price and discuss how these six factors affect the price for both European and American options
- Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks.
- Explain put-call parity and apply it to the valuation of European and American stock options.
- Explain the early exercise features of American call and put options.

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Identify the six factors that affect an option's price and discuss how these six factors affect the price for both European and American options

In the chart below, we show the directional impact of each input on the value of a call or put option, while all the other factors remain fixed.

Directional Impact of Each Input on Call or Put

Factor	Symbol	Call	Put
Stock price (↑)	S_0	↑+	↓ -
Strike price (↑)	K	↓ -	↑+
Time to expiration (↑)	T	↑+ (American) ⇔? (European)	↑+ (American) ⇔? (European)
Volatility (↑)	σ	↑+	↑+
Risk-free rate (↑)	r	↑+	↓ -
Div. yield (↑)	D	↓ -	↑+

Options are "volatility instruments"

Identify the six factors that affect an option's price and discuss how these six factors affect the price for both European and American options (continued)

With respect to each factor:

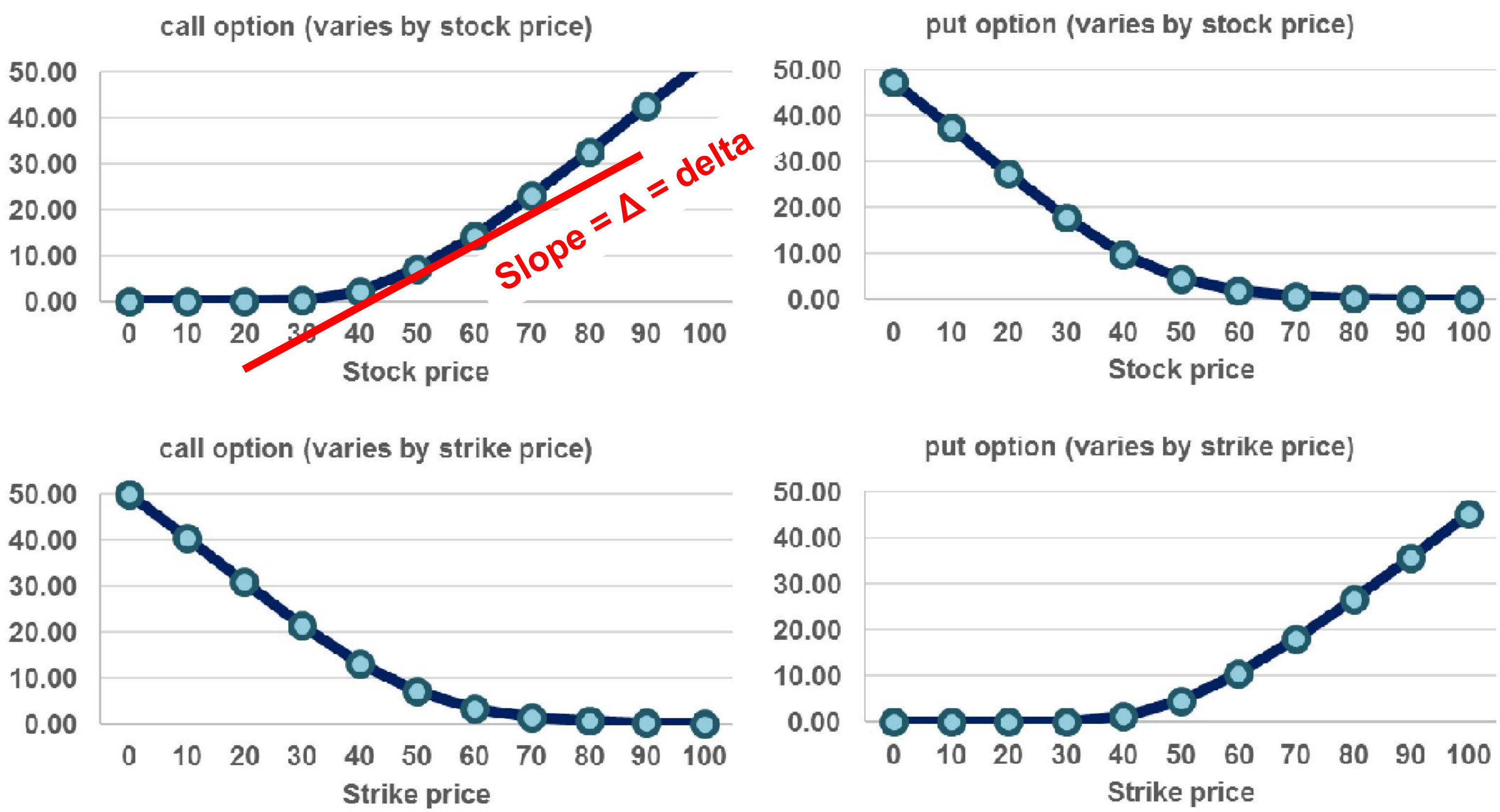
- **Stock price**: For a call option, a higher stock price implies greater intrinsic value
- **Strike price**: For a call option, a higher strike price implies less intrinsic value

- **Time to expiration**: For an American (call or put) option, option value is an **increasing function of greater time to expiration**. For a European option, while value will increase with greater time to expiration, the timing of dividends makes the relationship ambiguous (on dividend payout, the stock tends to drop).
- **Volatility**: Greater volatility increases value of both call and put option
- **Risk-free rate**: The minimum value of a European call option is the stock price minus the *discounted* strike price. A higher risk-free rate implies a lower discounted strike price and therefore, in turn, implies an increase in the value of the call option.

- **Dividend yield:** As dividends have the effect of reducing the stock price on the ex-dividend date, a higher dividend reduces the call option’s value.

Identify the six factors that affect an option's price and discuss how these six factors affect the price for both European and American options (continued)

These charts illustrate the effect of changes in stock price, strike price, expiration date, volatility and risk-free interest rate when S_0 = 50, K = 50, r_f=5%, σ= 30%, and T = 1.



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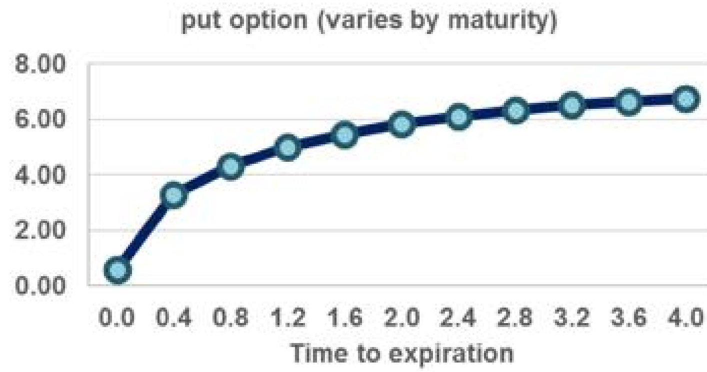

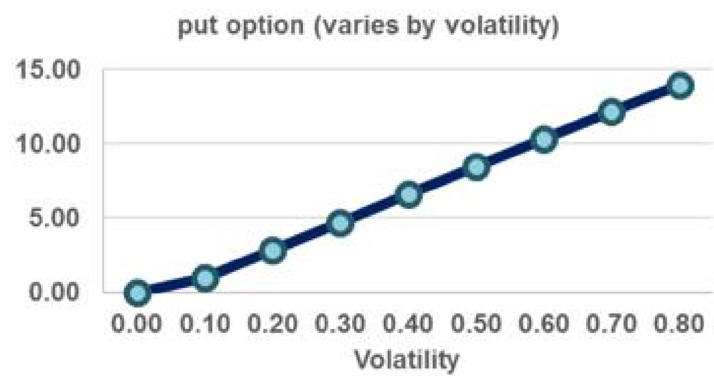

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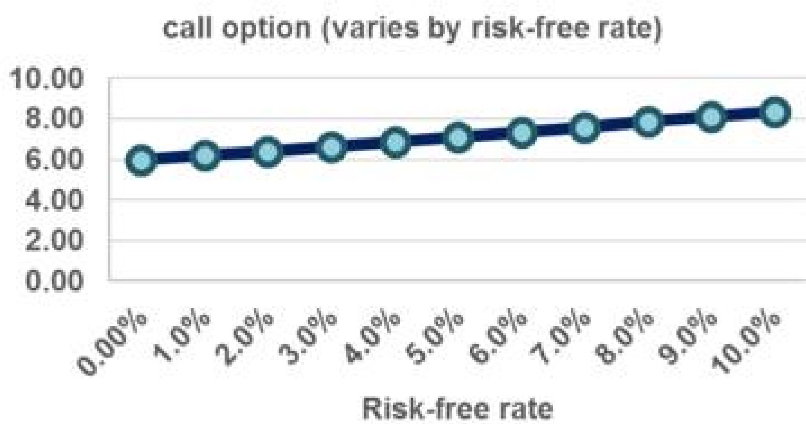

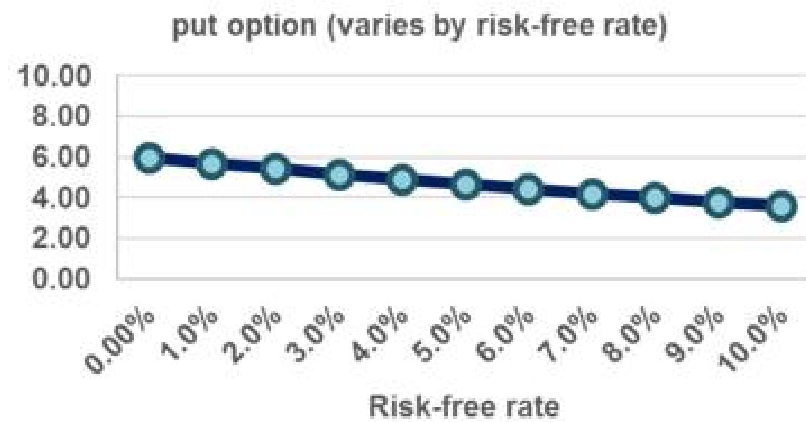

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Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks.

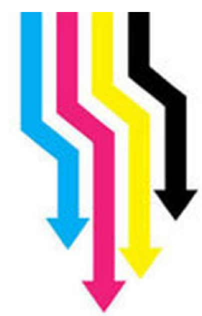
For options on **non-dividend paying stocks**, the boundaries are:

Upper Bound:

- An American or European call option gives the holder the right to buy one share of a stock for a certain price. No matter what happens, the option can never be worth more than the stock $(c \; or \; C \leq S_0)$
- An American put option gives the holder the right to sell one share of a stock for K. No matter how low the stock price becomes, the option can never be worth more than K. $(P \leq K)$
- For European put options, at maturity, the option cannot be worth more than K. It follows that it cannot be worth more than the present value of K today$(p \leq Ke^{-rT})$.

Lower Bound:

- A lower bound for the price of a European call option on a non-dividend-paying stock is the stock price minus discounted strike price. This is the price the Black-Scholes gives if the volatility input is equal to zero. Since the option can at worst expire worthless, it's value cannot negative, so lower bound is $c \geq max(S_0 - Ke^{-rT}, 0)$.
- For a European put option on a non-dividend-paying stock, lower bound is maximum of zero or discounted strike price minus the stock price $p \geq max\,(Ke^{-rT} - S_0, 0)$.



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Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks (cont)

For options on **dividend paying stocks**, if D denotes the present value of the dividends during the life of the option, then the lower boundaries show a change from that of options on non-dividend paying stocks to incorporate the effect of dividends.

The results are summarized in the table below. (Note the use of small ‘c’ and small ‘p’ for European options and capital ‘C’ and ‘P’ for American options).

Upper Bound	**European Call**	**American Call**	**European Put**	**American Put**
Div / Non-div stock	$\boldsymbol{c \leq S_0}$	$\boldsymbol{C \leq S_0}$	$\boldsymbol{p \leq Ke^{-rT}}$	$\boldsymbol{P \leq K}$
Lower Bound	**European Call**		**European Put**	
Non-div stock	$c \geq max(S_0 - Ke^{-rT}, 0)$		$p \geq \max(Ke^{-rT} - S_0, 0)$	
Div stock	$c \geq max(S_0 - D - Ke^{-rT}, 0)$		$p \geq \max(D + Ke^{-rT} - S_0, 0)$	

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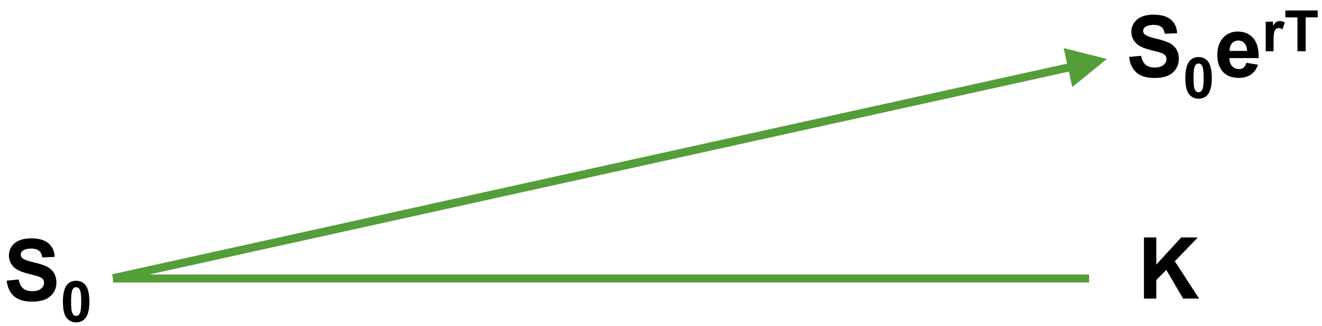
Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks (cont)

Intuition: minimum value of European call option



$$e^{-rT}(S_0e^{rT} – K) \quad \xleftarrow{\text{discounted}} \quad S_0e^{rT} - K$$

$$e^{-rT}\bullet S_0e^{rT} – Ke^{-rT}$$

$$S_0 – Ke^{-rT}$$

Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks (cont)

Hull Example 11.1

Consider a European call option on a non-dividend-paying stock where S_0= $51, the K = $50, T = 6 months (0.5), and r =12% per annum.

- The **upper bound** on the call option is $51: why would you pay more for an option than you could pay for the stock?
- The **lower bound** is the $max\left(51 - 50e^{-12\% \times 0.5}, 0\right)$ = $3.91. This is its so-called minimum value.

Hull Example 11.2

Consider a European put option on a non-dividend-paying stock when S_0= $38, K = $40, T = 3 months (0.25), and r = 10% per annum.

- As a put option cannot be worth more than its strike price at maturity, its **upper bound** is the present value of its strike price, 40$e^{-10\% \times 0.25}$ = $39.01.
- The **lower bound** is the $max\left(40e^{-12\% \times 0.25} - 38, 0\right)$ = $1.01. You would be willing to pay at least $1.01.

Explain put-call parity and apply it to the valuation of European and American stock options.

Put–call parity is based on a no-arbitrage argument; it can be shown that arbitrage opportunities exist if put–call parity does not hold. Put-call parity is given by:

$$c + Ke^{-rT} = p + S_0$$

For a European call with the same certain exercise price and exercise date as the European put, put–call parity shows that the value of the call combined with a bond which matures to yield the strike price of the option is equivalent to the value of a put plus a share of the stock.

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Explain put-call parity and apply it to the valuation of European and American stock options.

Please be ready to re-arrange put-call parity:

$$c = p + S_0 - Ke^{-rT}$$

$$p = c + Ke^{-rT} - S_0$$

$$c - p = S_0 - Ke^{-rT}$$

Note: Put-call parity only holds for European options. However, it is possible to derive some results for American option prices. It can be shown that, in case of American options, when there are no dividends,

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$$

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Explain put-call parity and apply it to the valuation of European and American stock options (continued)

Typical question: A typical application is to solve for the price of a call or put given the other variables. For example, assume we know that a one-year European put (p) is valued at \$2. If the risk-free rate is 4%, what is the value of the corresponding European call (i.e., one-year term) if the strike price is \$10 (K)and the stock price is \$11 (S_0)?

$$\boldsymbol{c = p + S_0 - Ke^{-rT} \Longrightarrow}$$
$$\boldsymbol{2 + 11 - 10e^{-4\% \times 1} = \$3.39}$$

Explain put-call parity and apply it to the valuation of European and American stock options (continued)

	European Call	European Put
Stock	**$51.00**	**$38.00**
Strike	**$50.00**	**$40.00**
Volatity	**20%**	**20%**
Riskfree rate	**12%**	**10%**
Term	**0.50**	**0.25**
Stock goes up to	**$60**	**$45**
Long call + bond	$60	$45
Protective put	$60	$45
Stock goes down to	**$40**	**$35**
Long call + bond	$50	$40
Protective put	$50	$40
Put-Call Parity		
Discounted Strike	$47.09	$39.01
Long call	$5.15	$1.08
$c + Ke^{-rT}$	**$52.24**	**$40.09**
Stock	$51.00	$38.00
Long put	$1.24	$2.09
$p + S_0$	**$52.24**	**$40.09**

Put-Call parity for European options

To illustrate put-call parity for the European call option (Hull Ex 11.1), assume two portfolios:

- The first portfolio is a call option combined with a $50 par bond
- The second portfolio is a put along with a share of stock priced at $51

Now consider the payoff of each portfolio if the stock increases to $60.

- The payoff on the first portfolio is $60: call gain of $10(=60-50) plus $50 bond
- The payoff on second portfolio is $60, the stock's price, as the put expires worthless.

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Explain put-call parity and apply it to the valuation of European and American stock options (continued)

	Ex 11.1	Ex 11.2
Stock	$51.00	$38.00
Strike	$50.00	$40.00
Volatity	20%	20%
Variance, σ^2	4.00%	4.00%
Riskfree rate	12%	10%
Term	0.50	0.25

Put-Call Parity		
Disc. Strike	$47.09	$39.01
Long call	$5.15	$1.08
$c + Ke^{-rT}$	$52.24	$40.09
Stock	$51.00	$38.00
Long put	$1.24	$2.09
$p + S_0$	$52.24	$40.09

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Explain put-call parity and apply it to the valuation of European and American stock options (continued)

Now consider the payoff of each portfolio if the stock drops to $40.

- The payoff on the first portfolio is the $50 bond, and the call expires worthless.
- The payoff on the second portfolio is $50: put gain of $10(=50-40) plus $40 stock.

Thus, the portfolios have the same payoff regardless of the stock price at maturity! Similarly, the put-call parity can be proved for the European put (Hull Ex 11.2).

Explain put-call parity and apply it to the valuation of European and American stock options (continued)

Put-Call parity for American options

- **Hull Ex 11.3:** An American call option on a non-dividend-paying stock with strike price $20.00 and maturity in 5 months is worth $1.50. Suppose that the current stock price is $19.00 and the risk-free interest rate is 10% per annum. For American options,

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$$

$$19 - 20 \leq C - P \leq 19 - 20e^{-10\% \times 5/12}$$

- From this equation and with C = $1.50, the upper and lower bounds for an American put with the same strike price and expiration date as the American call are $2.50 and $1.68.

Explain the early exercise features of American call and put options.

An American-style option can be exercised prior to expiration (i.e., can be exercised early). **A European option can only be exercised on the expiration date itself**.

All other things being equal, the value of an American style option must be at least as great as a European option with the same features:

$$\boldsymbol{Value\ [American\ option]\ \geq\ Value\ [European\ option]}$$

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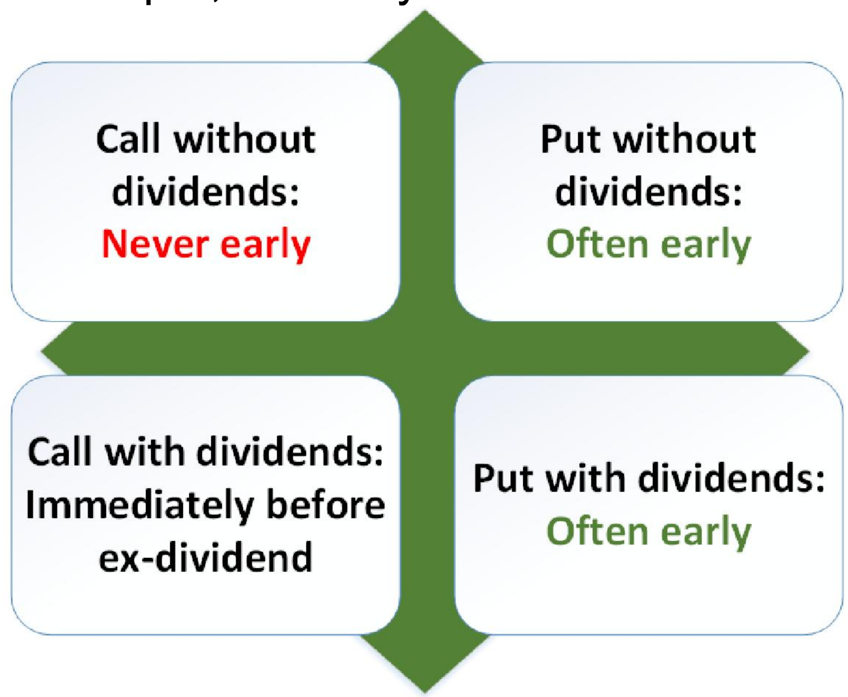
Explain the early exercise features of American call and put options (continued)

From a mathematical standpoint, **it is never optimal to execute an early exercise on an American call option on a non-dividend paying stock**. However, it can be optimal to execute an early exercise on an American put. In general, we can say that for an American put, the early exercise becomes more attractive as:

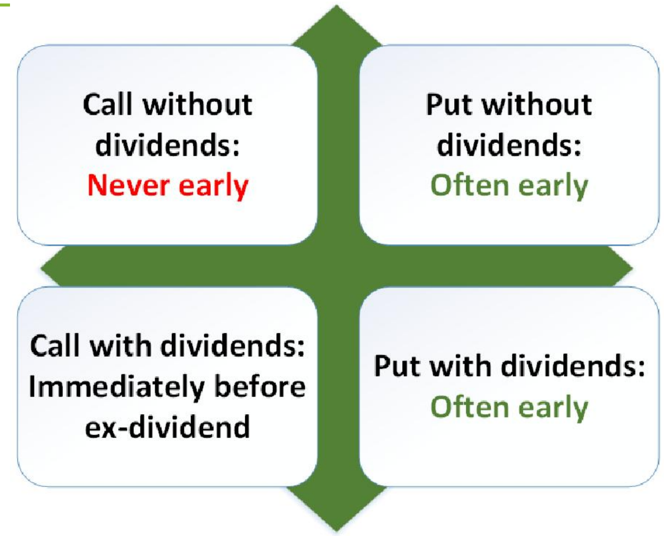
- Stock price (S_0) decreases,
- Risk-free (r) rate increases, and/or
- Volatility (σ) decreases.



Explain the early exercise features of American call and put options (continued)

American call should **not** be early exercised …

1. In effect provides “insurance” against stock price drop
2. Time value of money: delaying payment of fixed strike price



But it **can be optimal** to early exercise an American put

1. If put is in-the-money, limited opportunity further gains might be outweighed by realization of gain (the stock’s expected “drift” is upward)

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The End

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