P1.T3. Financial Markets & Products

Hull, Options, Futures & Other Derivatives
Exotic Options

Bionic Turtle FRM Video Tutorials

By David Harper, CFA FRM

Exotic Options

- Define and contrast exotic derivatives and plain vanilla derivatives.
- Describe some of the factors that drive the development of exotic products.
- Explain how any derivative can be converted into a zero-cost product.
- Describe how standard American options can be transformed into nonstandard American options.
- Identify and describe the characteristics and pay-off structure of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, shout, Asian, exchange, rainbow, and basket options.
- Describe and contrast volatility and variance swaps.
- Explain the basic premise of static option replication and how it can be applied to hedging exotic options.

Exotic Options

	Mechanics	Note
Forward start	*deferred*	**PV =** $\boldsymbol{ce^{-qT_1}}$
Compound	call/put on a call/put	
Chooser	choose max(c,p)	*path-dependent*
Barrier	Knock-in or knock-out at (h)	
Binary	Cash/Asset or Nothing “all-or-nothing”	Euro c = long binary asset – short binary cash
Lookback **floating** → **fixed** →	$\boldsymbol{S_T - S_{(MIN)}}$ $\boldsymbol{S_{(MAX)} - S_T}$ $\boldsymbol{S_{(MAX)} - K}$ $\boldsymbol{K - S_{(MIN)}}$	*path-dependent*
Shout	$\boldsymbol{MAX\ (0,\ S_T - S_1) + (S_1 - K)}$	*path-dependent*
Asian	$MAX\ [\,0, S_A - K]$ $MAX\ [\,0, K - S_A\,]$ $MAX\ [\,0, S_T - S_A\,]$ $MAX\ [\,0, S_A - S_T\,]$	*path-dependent*
Exchange	$\boldsymbol{MAX(V_T - U_T, 0)}$	

BIONIC TURTLE

Define and contrast exotic derivatives and plain vanilla derivatives.

Plain Vanilla **derivatives are the standard European and American call and put options traded on the exchanges.**

- They have standard well-defined properties and trade actively.
- Their prices or implied volatilities are quoted by exchanges or by brokers on a regular basis.

Exotic derivatives (aka, "exotics") **are non-standard options that are mostly traded over-the-counter.**

Exotic creature!

- They include a number of nonstandard products that have been created by financial engineers.
- Although they are a small part of the portfolio, exotics are important to a dealer because they much more profitable than vanilla products.

Describe some of the factors that drive the development of exotic products.

Exotic products are developed for a number of reasons.

- To meet a genuine hedging need in the market
- Sometimes there are **tax, accounting, legal, or regulatory** reasons why corporate treasurers, fund managers, and financial institutions find exotic products attractive
- Sometimes products are designed to reflect a view on potential future movements in particular market variables;
- Occasionally an exotic product is designed by an investment bank to appear more attractive than it is to an unwary corporate treasurer or fund manager.

BIONIC TURTLE

Explain how any derivative can be converted into a zero-cost product.

A package is a portfolio consisting of standard European calls, standard European puts, forward contracts, cash, and the underlying asset itself. **Often a package is structured by traders so that it has zero cost initially.**

Any derivative can be converted into a zero-cost product by deferring payment until maturity. For a European call option:

- Cost of the option when payment is made at time zero = c
- Cost when payment is made at maturity of the option (T) = A = ce^{rT}
- Payoff: $max\,(S_T - K, 0) - A$ or $max\,(S_T - K - A, -A)$.
- When the strike price, K, equals the forward price, deferred payment options are also known as break forward, Boston option, forward with optional exit, and cancelable forward.

Describe how standard American options can be transformed into nonstandard American options.

In a standard American option, "early exercise" can take place at any time during the life of the option and the exercise price is constant.

Such standard American options can be converted into nonstandard American options by:

- Bermudan option: Restricting early exercise to certain dates. (Bermuda is between Europe and America!)
- Allowing early exercise during only part of the life of the option. For example, there may be an initial "lock out" period with no early exercise.
- Changing the strike price during the life of the option.

Describe how standard American options can be transformed into nonstandard American options.

[… continued …]

Standard American options can be converted into nonstandard American options by:

- ❑ Bermudan option
- ❑ Allowing early exercise during only part of the life of the option.
- ❑ Changing the strike price during the life of the option.

Warrants issued by corporations on their own stock often have some/all of these features.

- For example, in a 7-year warrant, exercise might be possible on particular dates during years 3 to 7, with the strike price being $30 during years 3 and 4, $32 during the next 2 years, and $33 during the final year.

Such nonstandard American options can usually be valued using a binomial tree. At each node, the test (if any) for early exercise is adjusted to reflect the terms of the option.

BIONIC TURTLE

Identify and describe the characteristics and pay-off structure of: **Gap Options**

Gap Option

A **gap option** has a strike price of K_1
(which determines the payoff),
and a trigger price of K_2
(which determines whether option makes the payoff).

The strike price may be greater than or less than the trigger price. **If the trigger price is less than the strike price, then presumably the holder is *required to exercise*.**

For a given strike price, the trigger price that produces the maximum gap option price is the strike price.

- When strike price = trigger price, the gap option is the same as an ordinary option.
- *Increasing* or *decreasing* the trigger price above (or below) the strike price causes the relative value of the gap option to fall

BIONIC TURTLE

Identify and describe the characteristics and pay-off structure of: **Gap Options (continued)**

Gap Option

The ***trigger price*** determines whether or not the gap option has a nonzero payoff. The ***strike price*** determines the amount of the nonzero payoff.

- When the final stock price exceeds the trigger price, **gap call option** has a nonzero payoff (which may be positive or negative):

$$S_T - K_1 \text{ when } S_T > K_2$$

- A **gap put option** has a nonzero payoff if the final stock price is less than the trigger price:

$$K_1 - S_T \text{ when } S_T < K_2$$

BIONIC TURTLE

Identify and describe the characteristics and pay-off structure of: **Gap Options (continued)**

Gap Option

Gap options are valued with a small modification to BSM:

- The price for gap options are greater than the price given by the BSM for a regular call option (with strike price K_2) by: $K_2 - K_1 e^{-rt} N(d_2)$

	Gap option	Regular option
Call	$S_0 N(d_1) - K_1 e^{-rt} N(d_2)$	$S_0 N(d_1) - K e^{-rt} N(d_2)$
Put	$K_1 e^{-rt} N(-d_2) - S_0 N(-d_1)$	$K e^{-rt} N(-d_2) - S_0 N(-d_1)$

- For gap options: $d_1 = \frac{\ln(S_0/K_2)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}$; $d_2 = d_1 - \sigma\sqrt{T}$

BIONIC TURTLE

Identify and describe the characteristics and pay-off structure of: **Gap Options (continued)**

Gap Option

Hull Ex 26.1: An asset is currently worth \$500,000 (S_0). Over the next year, it is expected to have a volatility of 20%. The risk-free rate is 5%, and no income is expected. Suppose an insurance company has provided a **regular put option** where the policyholder has the right to sell the asset for \$400,000 (K)in one year if asset value has fallen below that level. This is a regular put option.

Suppose the cost of transferring the asset is \$50,000 and this is borne by the policyholder. The option is then exercised only if the value of the asset is less than \$350,000(K_2-trigger price). Here the strike price K_1 is still \$400,000 but the trigger price, K_2, is \$350.000. This is a **gap put option**.

Identify and describe the characteristics and pay-off structure of: **Gap Options (continued)**

Gap Option

	Regular Put	Gap put
Asset Price, S(0)	$500,000.00	$500,000.00
Strike Price, K or K1	K = $400,000.00	K1 = $400,000.00
Trigger Price, K2	N/A	K2 = $350,000.00
Volatility	20%	20%
Variance, σ^2	4.00%	4.00%
Riskfree rate	5.0%	5.0%
Term	1.00	1.00
-d1	-1.4657	-2.1334
N(-d1)	0.0714	0.0164
-d2	-1.2657	-1.9334
N(-d2)	0.1028	0.0266
Put Price	$3,435.95	$1,895.69

As shown in the table:

The value of the regular put option is $3,436 which is calculated from:

- $Ke^{-rt}N(-d_2) - S_0N(-d_1)$

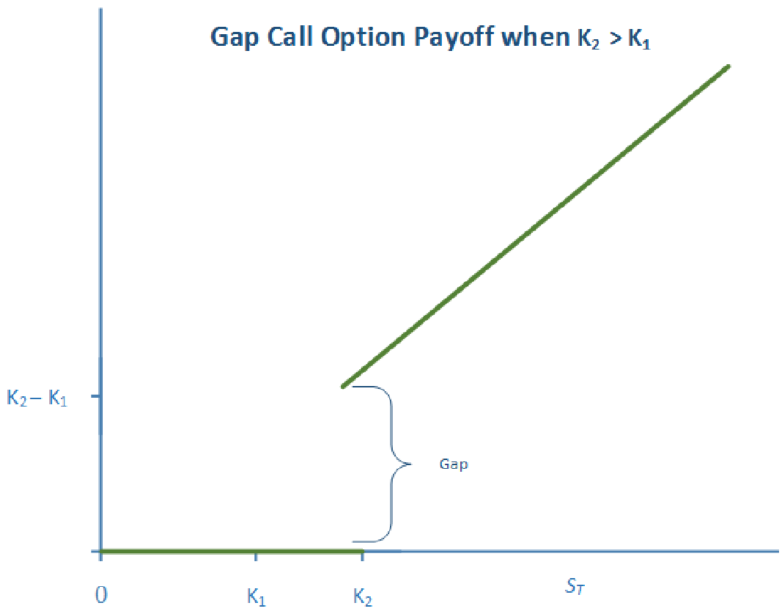
The value of the gap put option is $1,896 which is calculated from:

- $K_1e^{-rt}N(-d_2) - S_0N(-d_1)$ where $d_1 = \frac{\ln(S_0/K_2)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}$

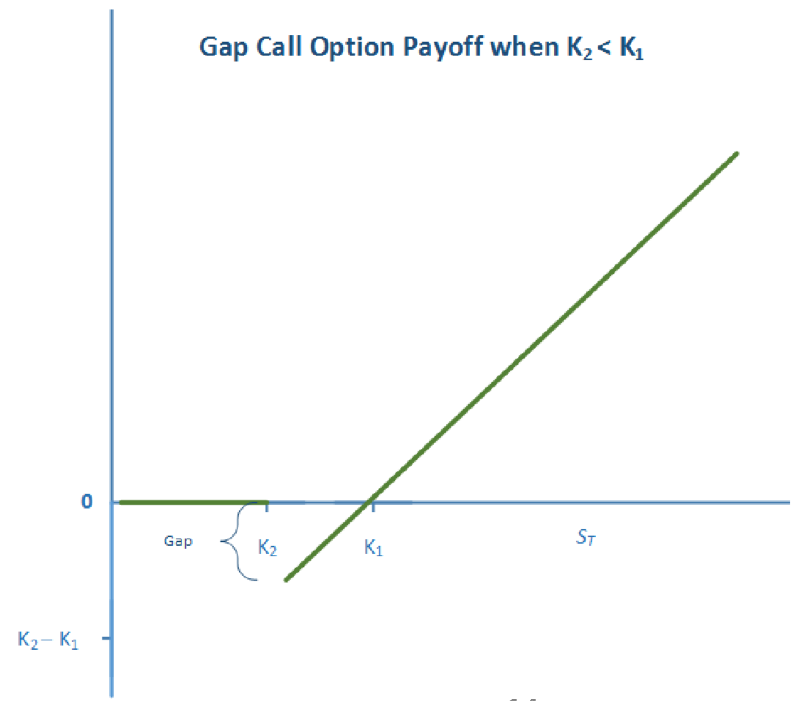
Identify and describe the characteristics and pay-off structure of: **Gap Options (continued)**

Gap Option

If we graph the payoff of a gap call option as a function of its final stock price, then we can see that there is a gap where $S_T = K_2$. In the graph below, there are no negative payoffs because the ***trigger price is greater than the strike price***:



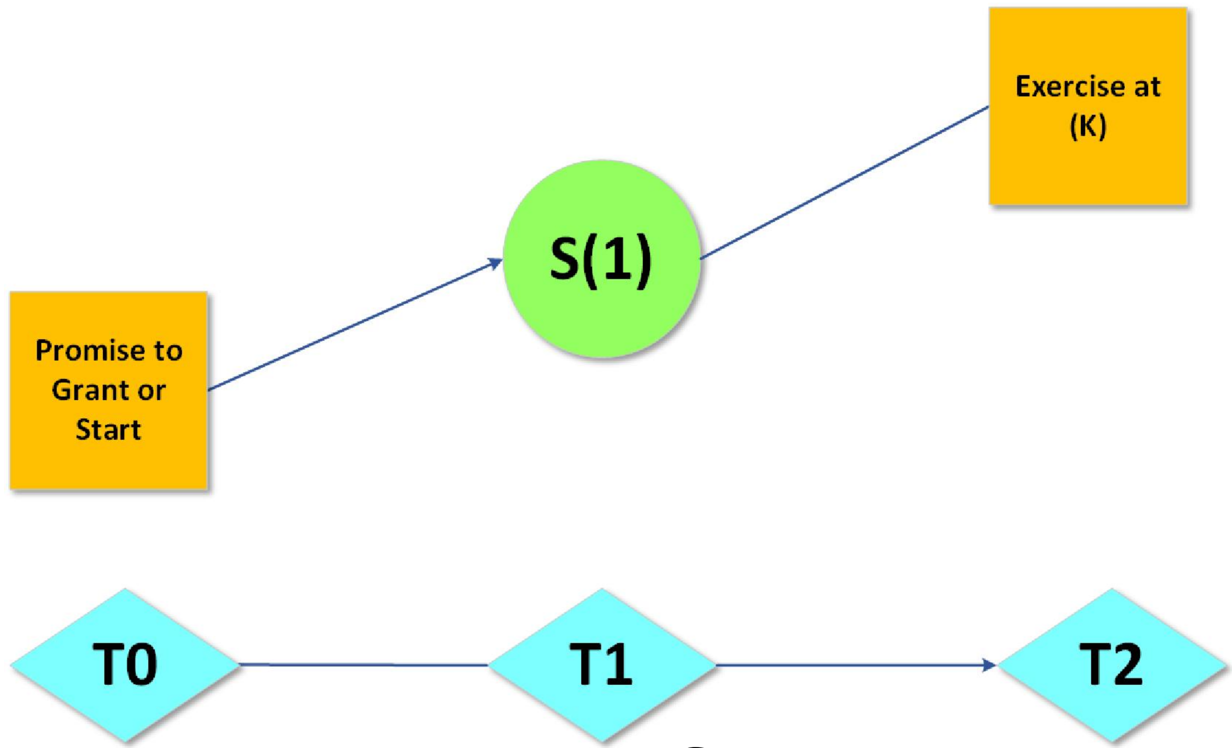
If the ***trigger price is less than the strike price*** for a gap call option, then negative payoffs are possible as shown below:



BIONIC TURTLE

Identify and describe the characteristics and pay-off structure of: **Forward start options**

Forward Start Options



$$ce^{-qT_1} \Leftarrow T_1 = c\frac{S_1}{S_0}$$

Identify and describe the characteristics and pay-off structure of: **Forward start options**

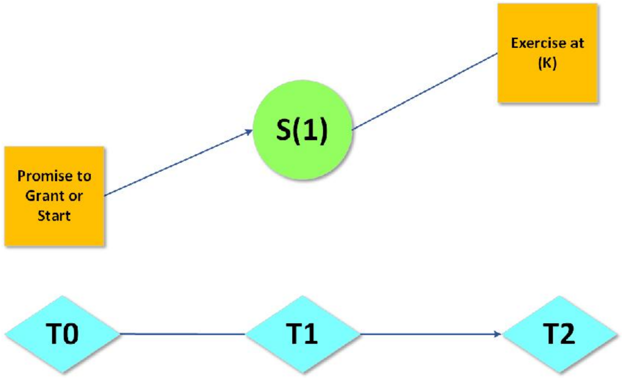
Forward Start Options

This slide skipped in video

Forward start options start at some time in the future. For eg. employee stock options, can be viewed as forward start options as the company commits (implicitly or explicitly) to granting at-the-money options to employees in the future.

The value of an at-the-money call option is proportional to the asset price. So, for an ***at-the-money forward start option***, starting at time T_1 and maturing at time T_2:

- **Its value at time T_1 is: $c\frac{S_1}{S_0}$**
- Its value today at T_o is given by today's value of the ATM option**:** ce^{-qT_1} where q is the dividend yield and c is the value, at time zero, of an ATM option with a life of $(T_2 - T_1)$.

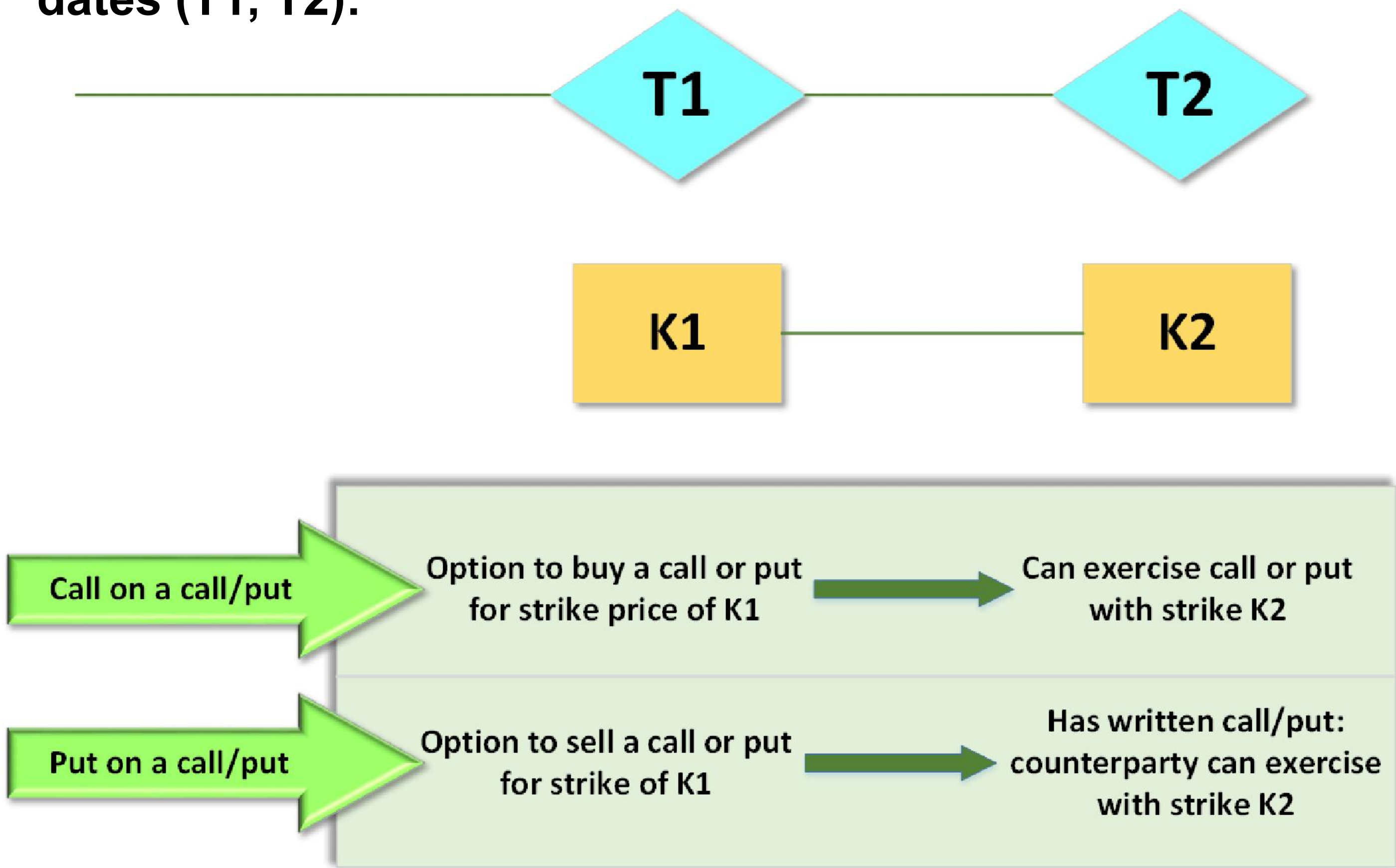


For a non-dividend-paying stock, q= 0 and the value of the forward start option is exactly the same as the value of a regular at-the-money option with the same life as the forward start option. For example, if the option value (as a percentage) is 20% of the stock price, and today's stock price is $20, then the value of the *forward start* ATM option is 20% * $20 = $4.00.

BIONIC TURTLE

Identify and describe the characteristics and pay-off structure of: **Compound options (continued)**

Compound options have two strikes (K1, K2) and two exercise dates (T1, T2):

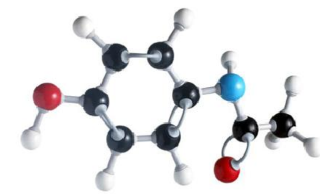


Identify and describe the characteristics and pay-off structure of: **Compound options**

Compound Options

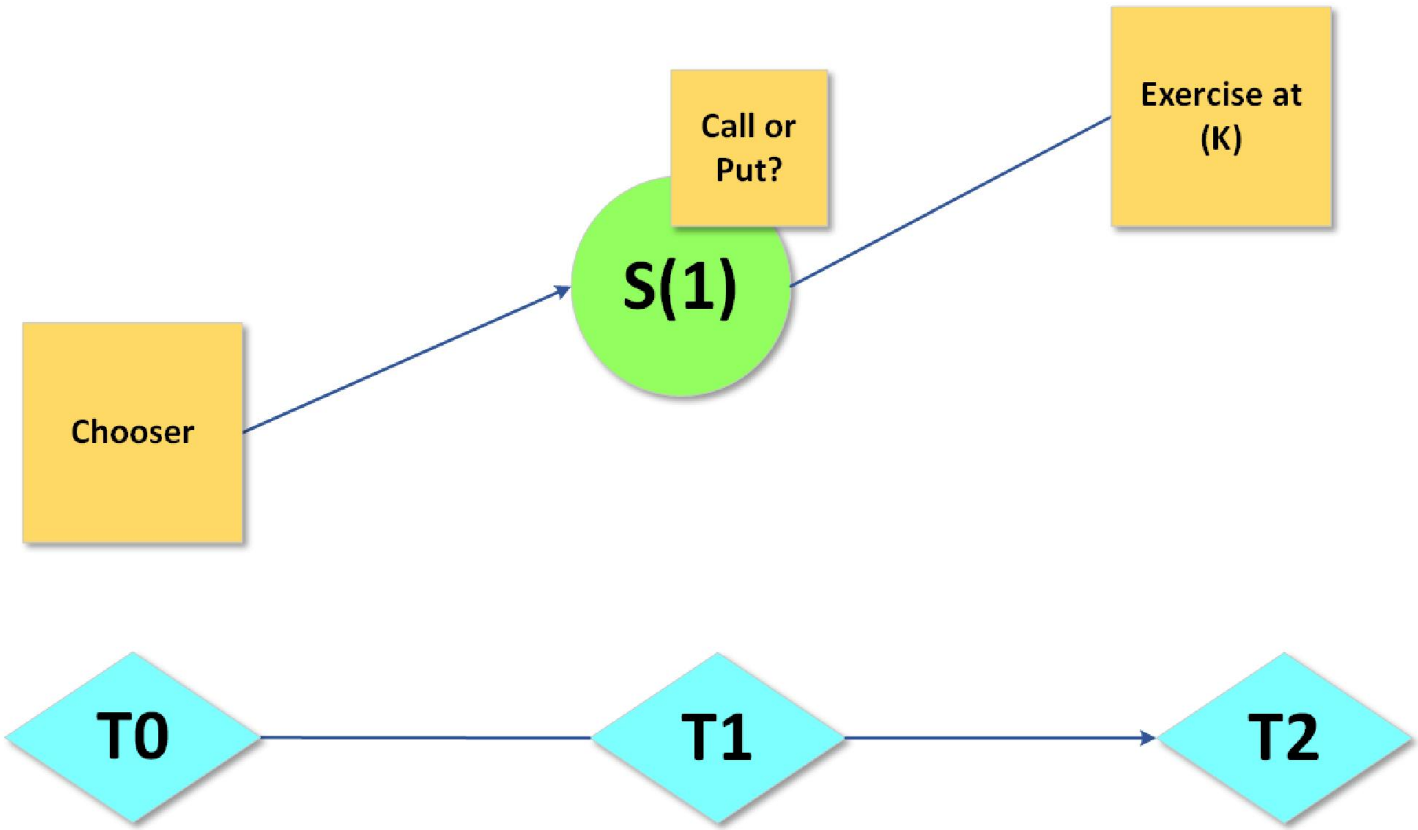
This slide skipped in video

Compound options are options on options. Compound options have two strike prices and two exercise dates. The four types are:



- **Call on a call**: option to pay K_1 at T_1, and receive a long option with strike K_2 at T_2
- **Call on a put**: option to pay K_1 at T_1, and receive a long put with strike K_2 at T_2
- **Put on a call**: option to sell (put) K_1 at T_1, a call option with strike K_2 at T_2; i.e., if exercised at T_1, holder will be short a call option
- **Put on a put**: option to sell (put) K_1 at T_1,, a put option with strike K_2 at T_2; i.e., if exercised at T_1, holder will be short a put option

Identify and describe the characteristics and pay-off structure of: **Chooser Options (continued)**

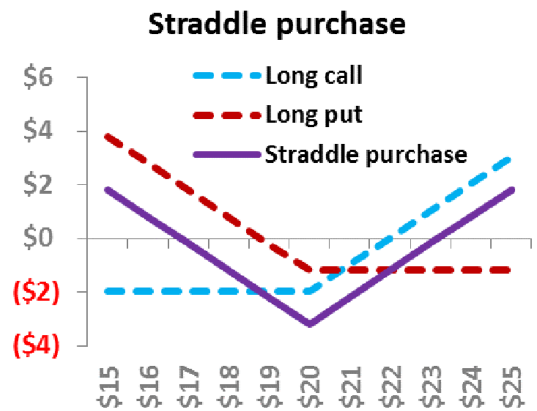


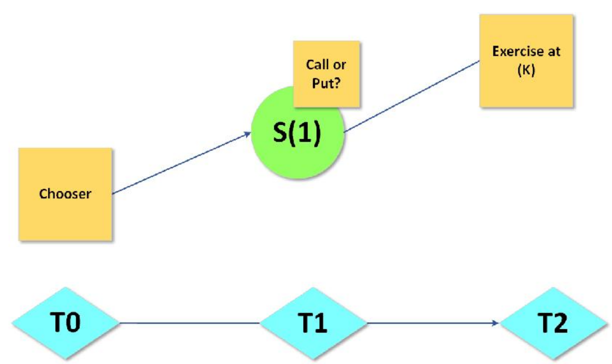
Identify and describe the characteristics and pay-off structure of: **Chooser Options (continued)**

Chooser Options

Compare a **chooser** to a straddle (long call + long put):

- Both are long volatility, but unsure about up/down direction.
- But investor in a chooser option is confident that the direction is revealed within a certain time frame and is therefore willing to choose between a put and a call after this time frame. As a result, the investor pays less for the chooser option than (s)he would have paid for the equivalent straddle.





Identify and describe the characteristics and pay-off structure of: **Chooser Options**

Chooser Options

This slide skipped in video

A chooser option (aka, "as you like it" option) gives the holder, after a specified period of time, the right to choose whether the option is a call or put.

The value of the chooser option at this point is $MAX\ (c, p)$.

If the options underlying the chooser option are both European and have the same strike price, put–call parity can be used to provide a valuation formula.
In this case, the chooser option is a package consisting of:

- A call option with strike price K and maturity T_2
- $e^{-q(T_2-T_1)}$ put options with strike price $Ke^{-(r-q)(T_2-T_1)}$ and maturity T_1

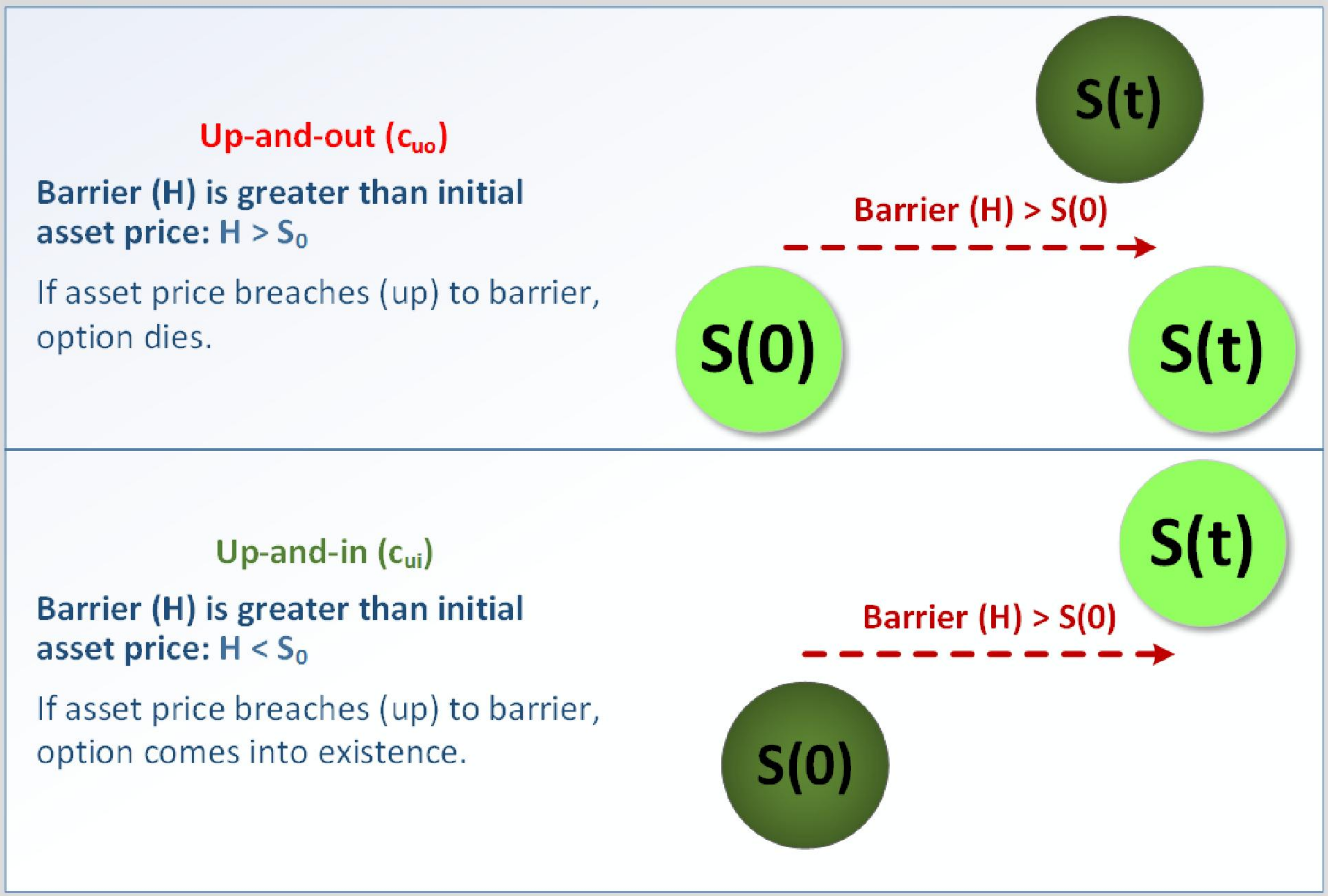
Identify and describe characteristics and pay-off structure of: **Barrier Options (continued)**

Four barrier option types: down/up and out/in

Identify and describe characteristics and pay-off structure of:
Barrier Options (continued)

Barrier Options

Four barrier option types: down/up and out/in



Up-and-out

Up-and-in

BIONIC TURTLE

Identify and describe characteristics and pay-off structure of: **Barrier Options**

Barrier Options

This slide skipped in video

A barrier option has a strike price (K) but additionally has a barrier price (H).

The barrier causes the option to be *cheaper than* the equivalent option without a barrier.

- **Knock-in barrier option:** if barrier breached, option comes into existence.
- **Knock-out barrier option**: if barrier breached, option ceases to exist.
- Down: **Barrier (H) is less than initial asset price:** $H < S_0$
- Up: **Barrier (H) is greater than initial asset price:** $H > S_0$

BIONIC TURTLE

Identify and describe characteristics and pay-off structure of:
Barrier Options (continued)

Properties of barrier options:

- The value of a regular options equals the value of a call (or put) down-and-in (up-and-in) plus the value of a call (or put) down-and-out (up-and-out) options:

  $$\boldsymbol{c = c_{do} + c_{di} \quad c = c_{uo} + c_{ui} \quad p = p_{do} + p_{di} \quad p = p_{uo} + p_{ui}}$$

  From this relation, the value of the barrier options can be derived.

- Note**:** In particular, when H is less than or equal to strike price K, the value of the up-and-out call, $\boldsymbol{c_{uo}}$ is zero and so, the value of the up-and-in call, $\boldsymbol{c_{ui} = c}$.

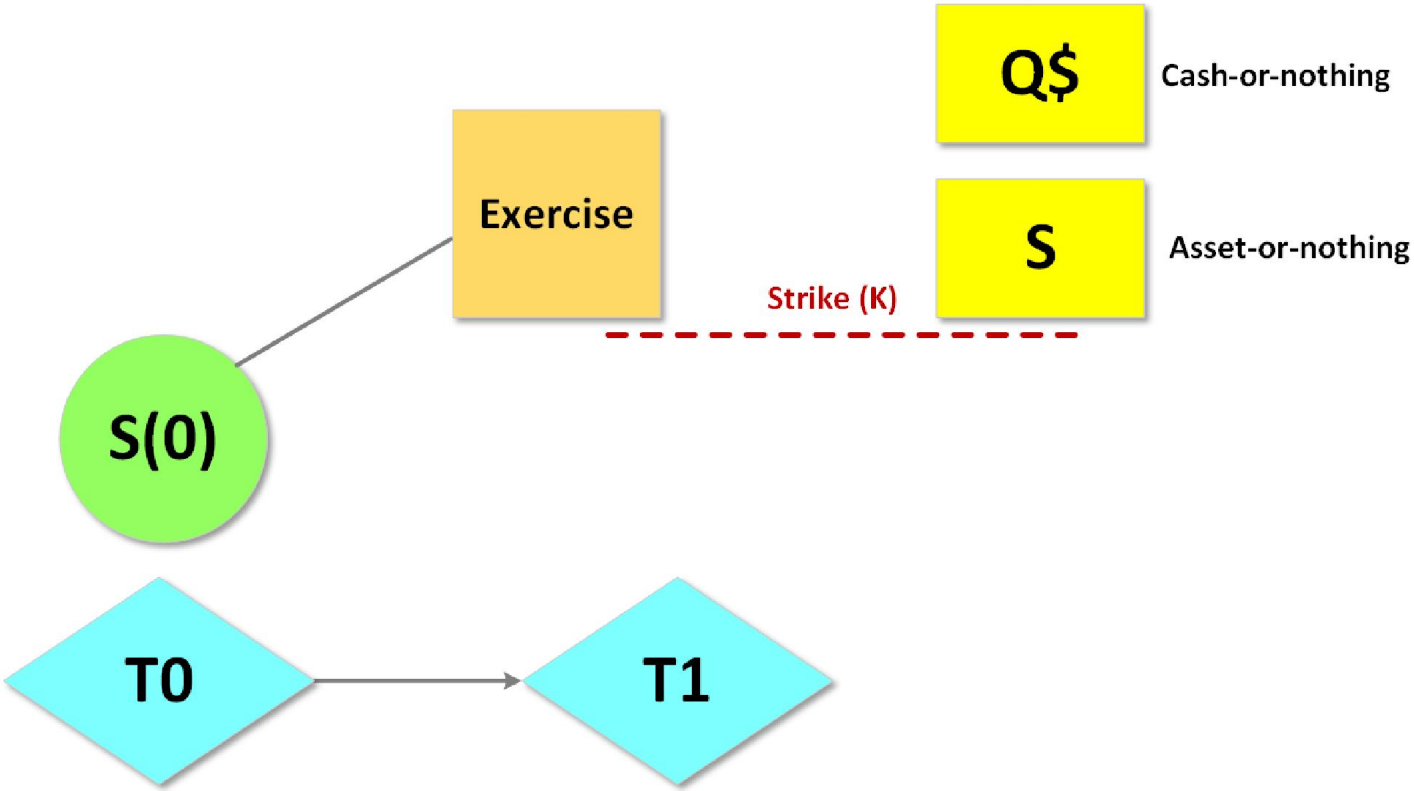
Identify and describe characteristics and pay-off structure of: **Barrier Options (continued)**

Properties of barrier options (continued):

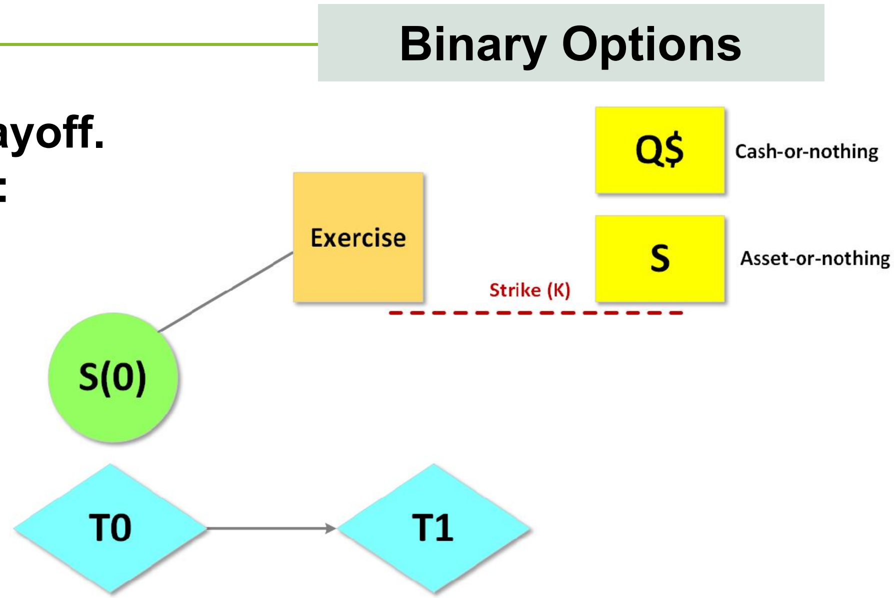
- As we increase the frequency with which we observe the asset price in determining whether the barrier has been crossed, the value of knock-out option goes down but the value of knock-in option goes up.
- Vega of barrier option can be negative (!)
- One disadvantage of the barrier options that a ‘‘spike’’ in the asset price can cause the option to be knocked in or out. An alternative structure is a **Parisian option**, where the asset price has to be above or below the barrier for a period of time for the option to be knocked in or out.

Identify and describe characteristics and pay-off structure of: **Binary options**

Binary options have discontinuous payoff.



Identify and describe characteristics and pay-off structure of:
Binary options



Binary options have discontinuous payoff. There are two kinds of binary options:

- **Cash-or-nothing**: Pays fixed (Q) or nothing.
- **Asset-or-nothing**: Pays asset price or nothing.

Value of:	Call	Put
Cash-or-nothing	$Qe^{-rt}N(d_2)$	$Qe^{-rt}N(-d_2)$
Asset-or-nothing	$S_0e^{-qt}N(d_1)$	$S_0e^{-qt}N(-d_1)$

BIONIC TURTLE

Identify and describe characteristics and pay-off structure of: **Binary options (continued)**

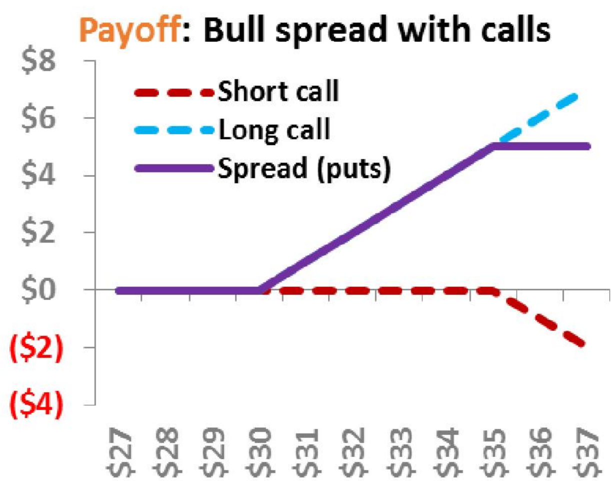
A binary option is similar to, but not the same as, a (bull) spread

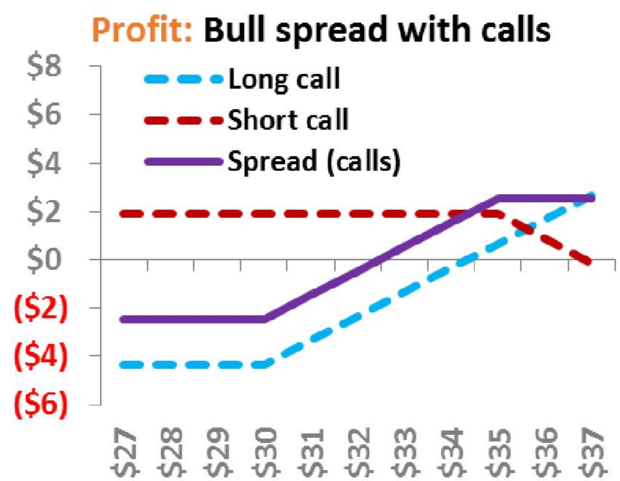
Consider an *all-or-nothing* binary call option with strike = $35 that pays $5 or zero

Compare to bull spread: Long call with strike @ $30 plus short call with strike @ $35.

Payoffs if stock prices at expiration is:

- ***From $30 to $35:*** Binary payoff = $0 and bull spread payoff = $1 to $5
- ***$35 and above:*** both their payoffs = $5





Identify and describe characteristics and pay-off structure of:

Binary options (continued)

European call option	Long asset-or-nothing call	
	Short cash-or-nothing call	

European put option	Long cash-or-nothing call	
	Short asset-or-nothing call	

"A regular European call option is equivalent to a long position in an asset-or-nothing call and a short position in a cash-or-nothing call where the cash payoff in the cash-or-nothing call equals the strike price. Similarly, a regular European put option is equivalent to a long position in a cash-or-nothing put and a short position in an asset-or-nothing put where the cash payoff on the cash-or-nothing put equals the strike price." – Hull

Identify and describe characteristics & pay-off structure of: **Lookback options**

Lookback Options

The payoffs from lookback options depend on the maximum or minimum asset price reached during the life of the option.

Lookback call options (floating and fixed):

A floating lookback call does not have a fixed strike price. Its payoff is the amount that final asset price exceeds the minimum asset price achieved during the life of the option.

Floating ("floating strike") lookback payoff = $S_T - S_{(MIN)}$

A fixed lookback call, on the other hand, does have a fixed strike price: Its payoff is the same as a regular European call option except the final asset price is replaced by the maximum asset price achieved during the life of the option.

Fixed ("fixed strike") lookback call payoff = $S_{(MAX)} - K$

Identify and describe characteristics & pay-off structure of: **Lookback options (continued)**

Lookback put options (floating and fixed):

A floating lookback put does not have a fixed strike price: its payoff is the amount by which the maximum asset price achieved during the life of the option exceeds the final asset price.

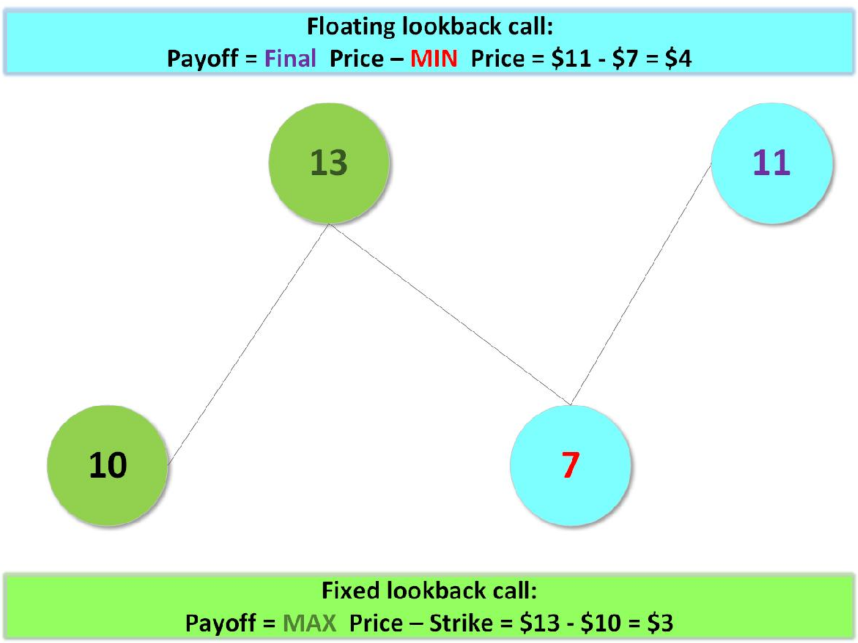
Floating lookback put payoff: $S_{(MAX)} - S_T$

A fixed lookback put, on the other hand, does have a fixed strike price: Its payoff is the same as a regular European put option except that the final asset price is replaced by the minimum asset price achieved during the life of the option.

Fixed lookback put payoff = $K - S_{(MIN)}$

Identify and describe characteristics & pay-off structure of: **Lookback options (continued)**

Consider a stock with initial price of $10 such that S_0= $10 which fluctuates up to S_1 = $13, down to S_2 = $7, and finally up to S_T= $11.



The payoff of the **floating lookback call** $4, the difference between the final price ($11) and the minimum price ($7).

In the case of the **fixed lookback call** if strike price K = $10, the payoff is the maximum ($13) minus strike price ($10).

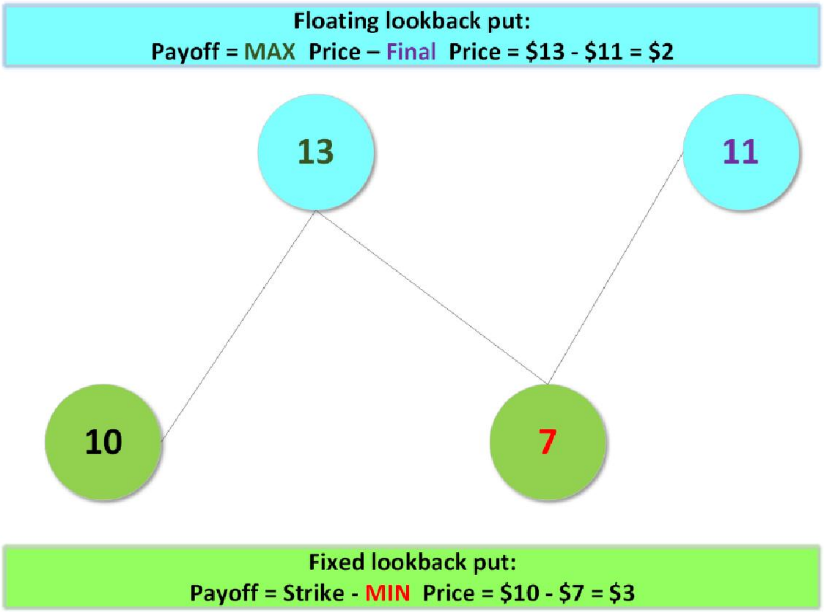
Identify and describe characteristics & pay-off structure of:
Lookback options (continued)

Lookback Options

Consider a stock with initial price of $10 such that S_0= $10 which fluctuates up to S_1 = $13, down to S_2 = $7, and finally up to S_T= $11.

Payoff of the **floating lookback put** is $2, the difference between maximum price ($13) and the final price ($11).

In the case of the **fixed lookback put** payoff is $3, difference between strike price K = $10 and minimum price ($7).

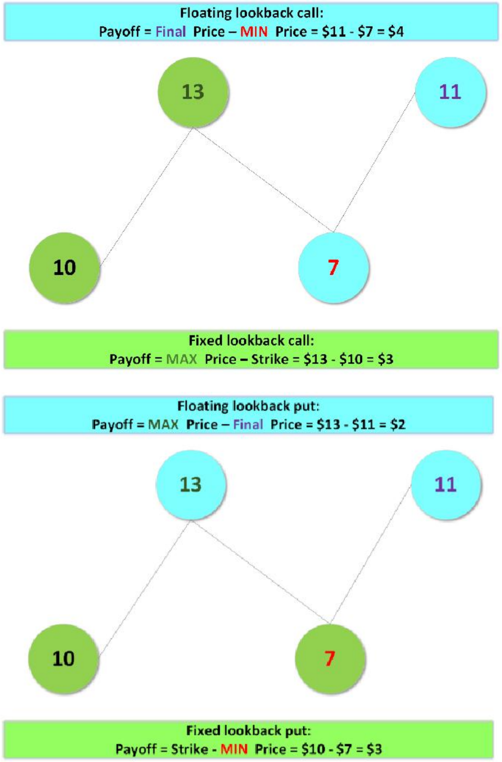


Identify and describe characteristics & pay-off structure of: **Lookback options (continued)**

Lookback Options

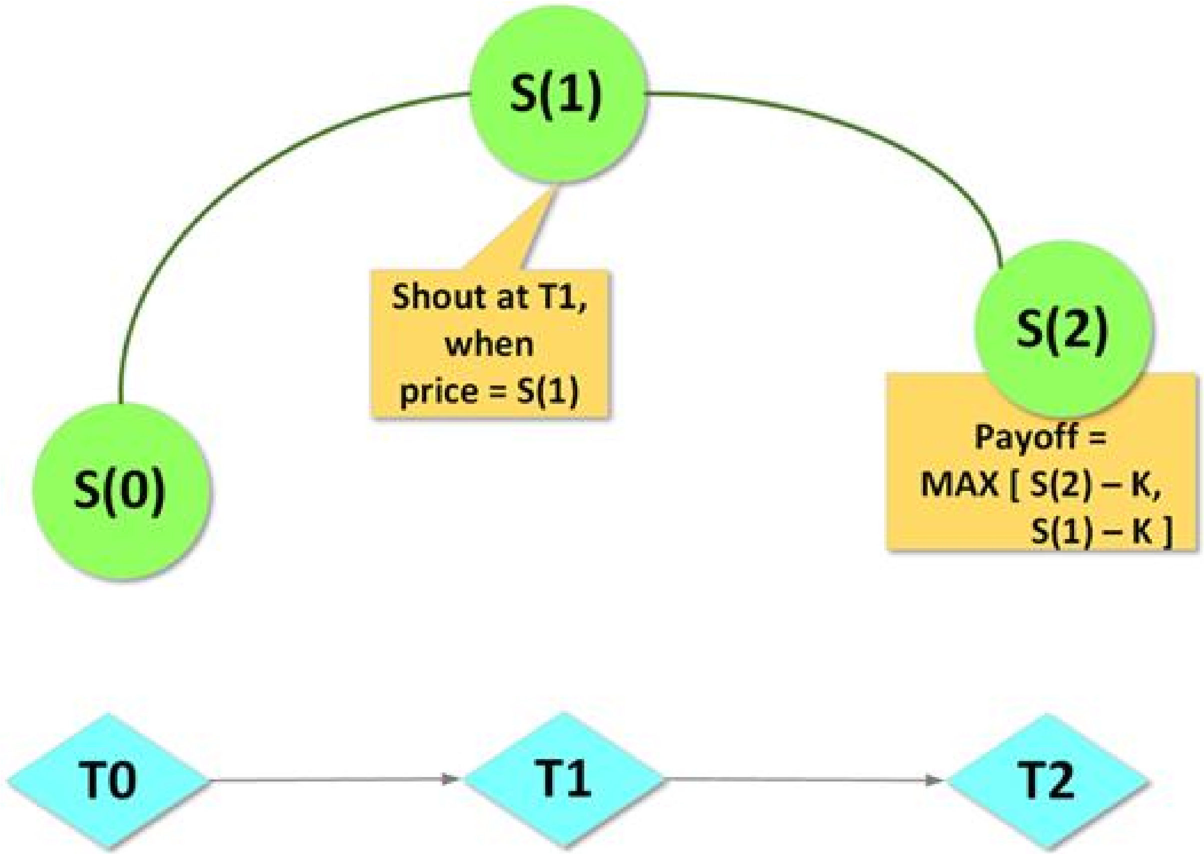
Properties:

- Lookbacks are appealing to investors, but very expensive when compared with regular options.
- As with barrier options, the value of a lookback option is liable to be sensitive to the frequency with which the asset price is observed for the purposes of computing the maximum or minimum.



BIONIC TURTLE

Identify and describe characteristics & pay-off structure of: **Shout Options**



Identify and describe characteristics & pay-off structure of:
Shout Options (continued)

Shout Options

For example, assume the strike price is $50 and the holder of a call **shouts** when the price of the underlying asset is $60. If the final asset price is less than $60, the holder receives a payoff of $10. If it is greater than $60, the holder receives the excess of the asset price over $50.

Features:

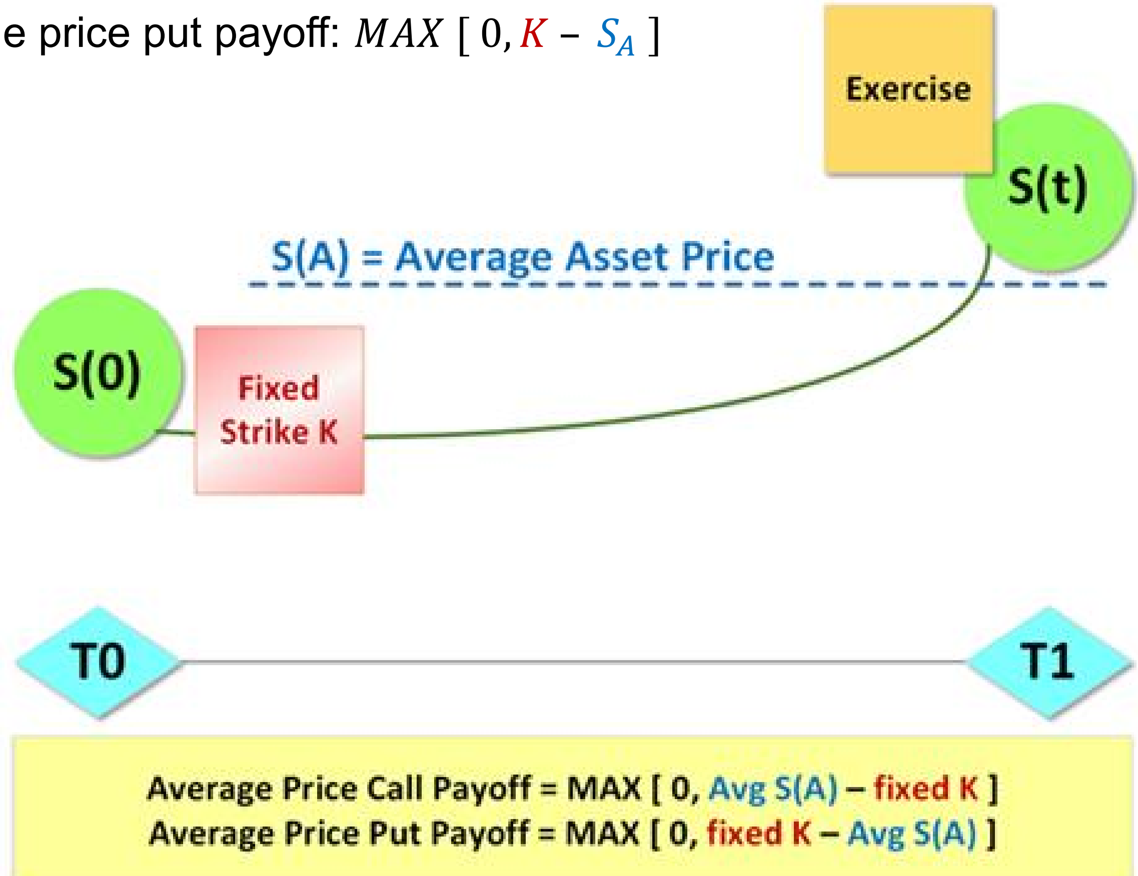
- A shout option has some of the same features as a lookback option, but is considerably less expensive.
- If the holder shouts at a time T_1 when the asset price is S_1 the payoff from the option is $MAX\ (0, S_T - S_1) + (S_1 - K)$

Identify and describe characteristics & pay-off structure of: **Asian Options**

Asian Options

Asian option payoff depends on average price (S_A) of the underlying asset during the life of the option.

- Average price call payoff: $MAX\ [\ 0, S_A - K]$
- Average price put payoff: $MAX\ [\ 0, K - S_A\ ]$

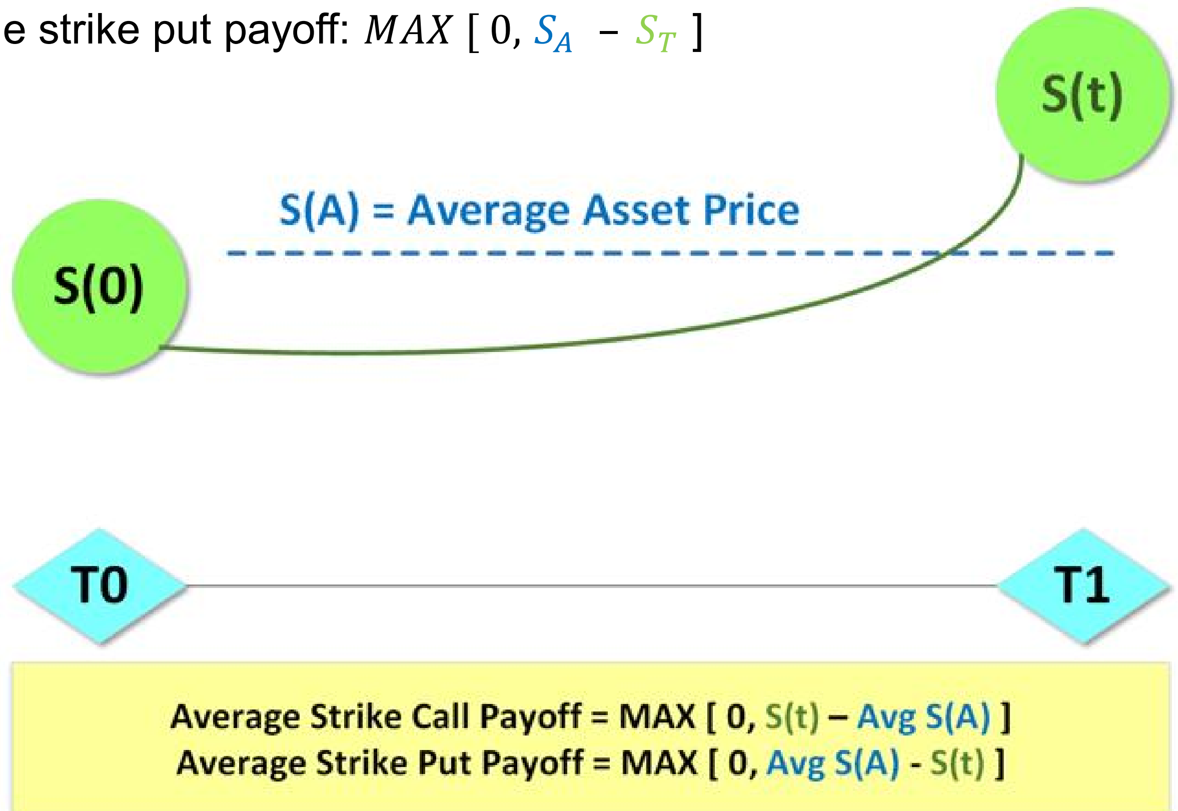


Identify and describe characteristics & pay-off structure of:
Asian Options (continued)

Asian Options

Another type of Asian options is an average strike option.

- Average strike call payoff: $MAX\ [\ 0,\ S_T\ -\ S_A\ ]$
- Average strike put payoff: $MAX\ [\ 0,\ S_A\ -\ S_T\ ]$



Identify and describe characteristics & pay-off structure of: **Asian Options (continued)**

Features:

- Average price options are less expensive than regular options. Average price options are also more appropriate than regular options for meeting certain needs of investors. For eg. a treasurer expecting to receive a cash flow in a foreign currency spread evenly over the next year will look for an option that guarantees that the average exchange rate realized during the year is above some level.
- Average strike options can guarantee that the average price paid for an asset in frequent trading over a period of time is not greater than the final price (or average price received is not less than the final price)

Identify and describe characteristics & pay-off structure of: **Exchange Options**

Exchange Options

Exchange options are options to exchange one asset for another. They arise in various contexts. For example,

- An option to buy yen with Australian dollars is, from the point of view of a US investor, an option to exchange one foreign currency asset for another foreign currency asset.
- A stock tender offer is an option to exchange shares in one stock for shares in another stock.

Consider a European option to give up an asset worth U_T at time T and receive in return an asset worth V_T. The payoff from this exchange option is:

$$\boldsymbol{max(V_T - U_T, 0)}$$

Identify and describe characteristics & pay-off structure of: **Exchange Options (continued)**

An exchange option can be valued with Black-Scholes variation called Margrabe.

- Suppose that the asset prices, U, V, both follow geometric Brownian motion with volatilities σ_U and σ_V. Suppose further that the instantaneous correlation between U and V is ρ, and the yields provided are q_U and q_V, respectively. U_0 and V_0 are the values of U and V at times zero. The value of the option at time zero is:

$$\boldsymbol{V_0 e^{-q_V T} N(d_1) - U_0 e^{-q_U T} N(d_2)}$$

where $\boldsymbol{d_1 = \frac{1n(V_0/U_0) + q_U - q_V + \hat{\sigma}^2/2)T}{\hat{\sigma}\sqrt{T}}}$, $\boldsymbol{d_2 = d_1 - \hat{\sigma}\sqrt{T}}$

$$\boldsymbol{\hat{\sigma} = \sqrt{\sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V}}$$

- This option price is the same as the price of U_0 European call options on an asset worth V/U when the strike price is 1.0, the risk-free interest rate is q_U , and the dividend yield on the asset is q_V .

BIONIC TURTLE

Identify and describe characteristics & pay-off structure of: **Exchange Options (continued)**

Exchange Options

An option to obtain **better or worse of two assets** can be regarded as a position in one of the assets combined with an option to exchange it for the other asset:

- $\boldsymbol{min(U_T, V_T) = V_T - max(V_T - U_T, 0)}$
- $\boldsymbol{max(U_T, V_T) = U_T + max(V_T - U_T, 0)}$

BIONIC TURTLE

Identify and describe characteristics & pay-off structure of:
Rainbow Options & Basket Options

Rainbow & Basket Options

Rainbow options involve two or more risky assets.

- For example, in a bond futures contract, party with short position is allowed to choose between a large number of different bonds when making delivery

The option involving several assets is a basket option.

- This is an option where the payoff is dependent on the value of a portfolio (or basket) of assets.
- The assets are usually either individual stocks or stock indices or currencies.
- A European basket option can be valued with Monte Carlo simulation, by assuming that the assets follow correlated geometric Brownian motion processes.

BIONIC TURTLE

Describe and contrast volatility and variance swaps.

Volatility swap is an agreement to exchange the **realized volatility** of an asset between time 0 and time T for a pre-specified volatility.

- Suppose that there are n daily observations on the asset price durir between time 0 and time T. The realized volatility is

$$\bar{\sigma} = \sqrt{\frac{252}{n-2}\sum_{i=1}^{n-1}\left[ln\left(\frac{S_i+1}{S_i}\right)\right]^2}$$

  where S_i is the *i*th observation on the asset price.

- The payoff from the volatility swap at time T to the payer of the fixed volatility is $L_{vol}(\bar{\sigma} - \sigma_K)$, where L_{vol} is the notional principal and σ_K is the fixed volatility.

- Whereas an option provides a complex exposure to the asset price and volatility, a volatility swap is simpler in that it has exposure only to volatility.

Describe and contrast volatility and variance swaps (continued)

Variance swap is an agreement to exchange the realized variance rate of an asset between time 0 and time T for a pre-specified variance.

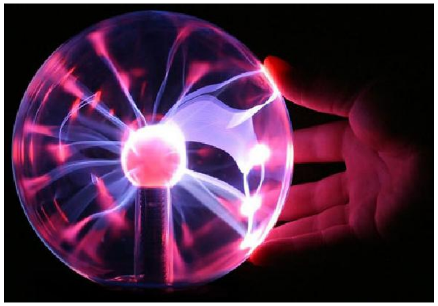
- The variance rate is the square of the volatility ($\bar{V} = \bar{\sigma}^2$)
- Variance swaps are easier to value because the variance rate can be replicated using a portfolio of puts and calls.
- The payoff from a variance swap at time T to the payer of the fixed variant rate is $L_{var}(\bar{V} - V_K)$, where L_{var} is the notional principal and V_K is the fixed variance rate.
- Often the notional principal for a variance swap is expressed in terms of the corresponding notional principal for a volatility swap using $L_{var} = L_{vol}/(2\sigma_K)$.

BIONIC TURTLE

Explain the basic premise of static option replication and how it can be applied to hedging exotic options.

Static Options

Static option replication involves searching for a portfolio of actively traded options that approximately replicates the exotic option. Shorting this position provides the hedge.



The basic principle underlying static options replication is as follows:

- If two portfolios are worth the same on a certain boundary, they are also worth the same at all interior points of the boundary.
- We can approach the hedging of exotic options by creating a delta neutral position; and rebalancing frequently to maintain delta neutrality.

Explain the basic premise of static option replication and how it can be applied to hedging exotic options (continued)

Some exotic options are easier to hedge than plain vanilla options and some are more difficult. For example:

Relatively Easy to Hedge

- An **average price option** where the averaging period is the whole life of the option. As time passes, we observe more of the asset prices that will be used in calculating the final average. This means that our uncertainty about the payoff decreases with the passage of time. As a result, the option becomes progressively easier to hedge. In-the final few days, the delta of the option always approaches zero because price movements during this time have very little impact on the payoff.

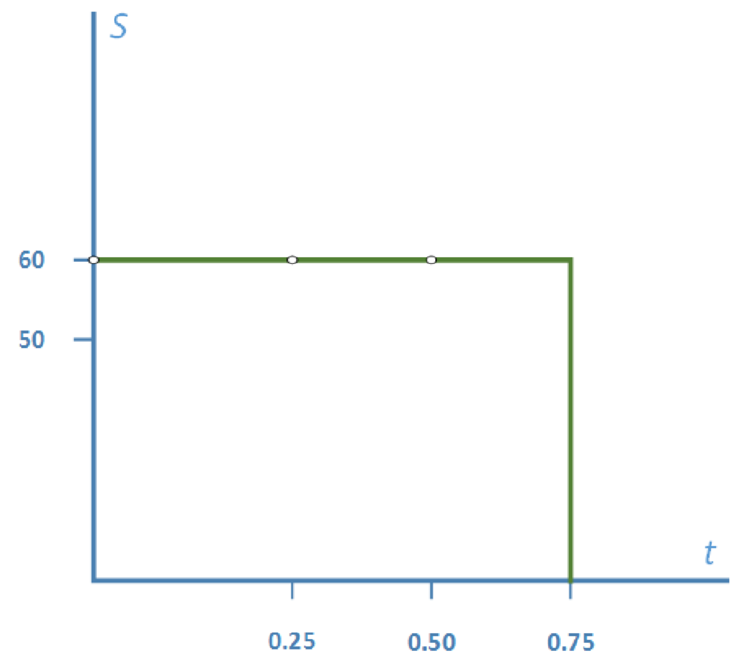
Relatively Difficult to Hedge

- **Barrier options**: Consider a down-and out call option on a currency when the exchange rate is 0.0005 above the barrier. If the barrier is hit, the option is worth nothing. If the barrier is not hit, the option may prove to be quite valuable. The delta of the option is discontinuous at the barrier, which makes conventional hedging very difficult.

Explain the basic premise of static option replication and how it can be applied to hedging exotic options (continued)

Example: Consider a 9-month up-and-out call option on a non-dividend-paying stock where the stock price is 50, the strike price is 50, the barrier is 60, the risk-free interest rate is 10% per annum, and the volatility is 30% per annum.

Suppose that $f\ (S, t)$ is the value of the option at time t for a stock price of S. Any boundary in (S, t) space can be used for the purposes of producing the replicating portfolio.

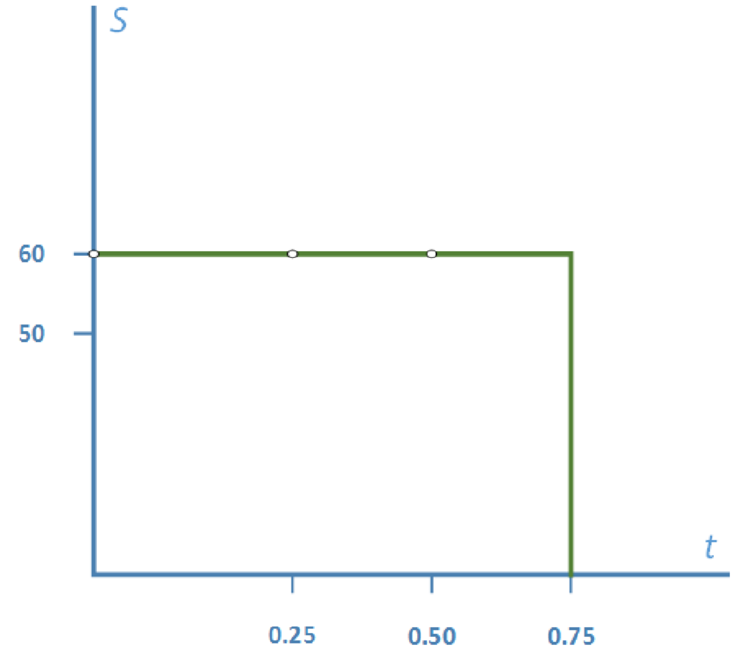


Explain the basic premise of static option replication and how it can be applied to hedging exotic options (continued)

As seen from the figure here, the replicating portfolio chosen is defined by S = 60 and t = 0.75. The boundary values of the up-and-out option are:

$$f(S, 0.75) = \max(S - 50, 0) \text{ when } S < 60$$

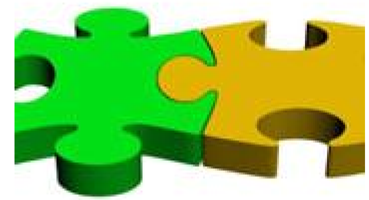
$$f(60, t) = 0 \text{ when } 0 \leq t \leq 0.75$$



BIONIC TURTLE

Explain the basic premise of static option replication and how it can be applied to hedging exotic options (continued)

There are many ways that these boundary values can be approximately matched using regular options. The natural option to match the first boundary is a 9-month European call with a strike price of 50. The first component of the replicating portfolio is therefore one unit of this option. (We refer to this option as option A.)



One way of matching the $f(60, t)$ boundary is to proceed as follows:

- Divide the life of the option into N steps of length Δt
- Choose a European call option with a strike price of 60 and maturity at time $N\Delta t$ (=9 months) to match the boundary at the $\{60, (N-1)\ \Delta t\}$ point.
- Choose a European call option with strike of 60 and maturity at time $(N-1)\Delta t$ to match the boundary at the $\{60, (N-2)\ \Delta t\}$ point and so on.
- Note that the options are chosen in sequence so that they have zero value on the parts of the boundary matched by earlier options.
- The option with a strike price of 60 that matures in 9 months has zero value on the vertical boundary that is matched by option A.
- The option maturing at time $i\Delta t$ has zero value at the point $\{60, i\ \Delta t\}$ that is matched by the option maturing at time $(i+1)\Delta t$ for $1 \le i \le N-1$.

Explain the basic premise of static option replication and how it can be applied to hedging exotic options (continued)

Suppose that Δ = 0.25. In addition to option A, the replicating portfolio consists of positions in European options with strike price 60 that mature in 9, 6, and 3 months. We will refer to these as options B, C, and D, respectively.

- Given our assumptions about volatility and interest rates, option B is worth 4.33 at the {60, 0.5} point. Option A is worth 11.54 at this point. The position in option B necessary to match the boundary at the {60, 0.51} point is: -11.54/4.33 = -2.66.
- Option C is worth 4.33 at the {60, 0.25} point. The position taken in options A and B is worth -4.21 at this point. The position in option C necessary to match the boundary at the {60, 0.25} point is: 4.21/4.33 = 0.97.

Explain the basic premise of static option replication and how it can be applied to hedging exotic options (continued)

- Similar calculations show that the position in option D necessary to match the boundary at the {60, 0} point is 0.28. The portfolio chosen is summarized in the table here.
- The portfolio is worth 0.73 initially (i.e., at time zero when the stock price is 50). This compares with 0.31 given by the analytic formula for the up-and-out call.

Option	Strike Price	Maturity (years)	Position	Initial Value
A	50	0.75	1	6.99
B	60	0.75	-2.66	-8.21
C	60	0.50	0.97	1.78
D	60	0.50	0.28	0.17
			Sum	0.73

- The replicating portfolio is not exactly the same as the up-and-out option because it matches the latter at only three points on the second boundary. If 18 points are matched the value reduces to 0.38 and if 100 points are matched, the value reduces further to 0.32.
- To hedge a derivative, the portfolio that replicates its boundary conditions must be shorted. The portfolio must be unwound when any part of the boundary is reached.
- **Static options replication has the advantage over delta hedging that it does not require frequent rebalancing.** It can be used for a wide range of derivatives. The user has a great deal of flexibility in choosing the boundary that is to be matched and the options that are to be used.

BIONIC TURTLE

The End

P1.T3. Financial Markets & Products

Hull, Options, Futures & Other Derivatives
Exotic Options