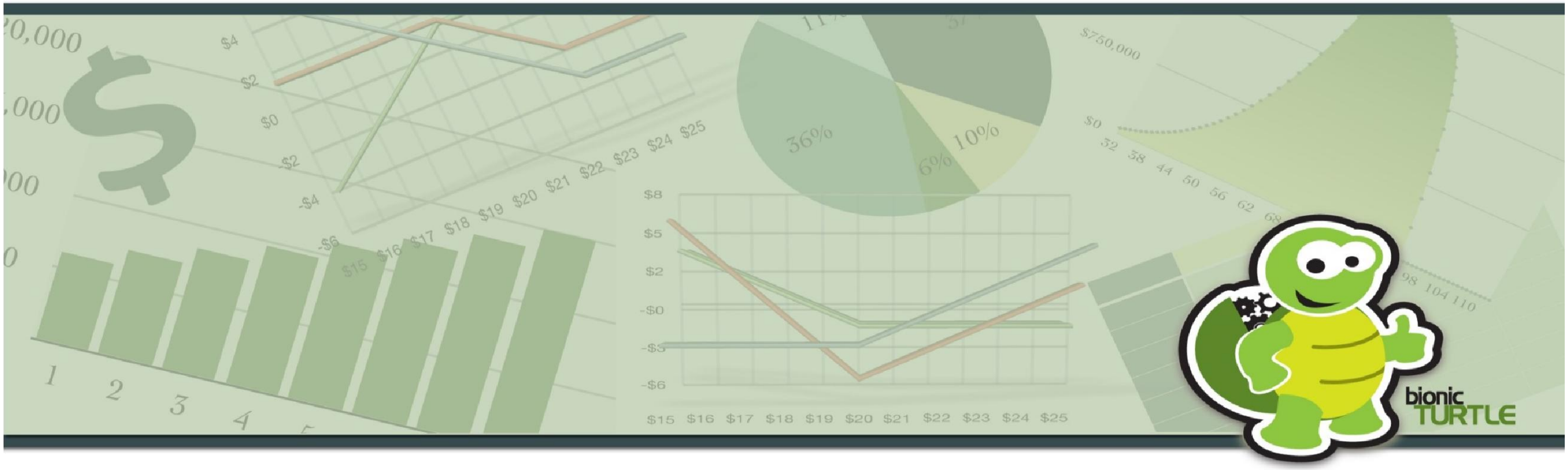


P1.T3. Financial Markets & Products

Hull, Options, Futures & Other Derivatives
Interest Rate Futures

Bionic Turtle FRM Video Tutorials

By David Harper, CFA FRM

BIONIC TURTLE

Interest Rate Futures

- Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.
- Calculate the conversion of a discount rate to a price for a U.S. Treasury bill.
- Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond.
- Explain and calculate a US Treasury bond futures contract conversion factor.
- Calculate the cost of delivering a bond into a Treasury bond Futures contract.
- Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision.
- Calculate the theoretical futures price for a Treasury bond futures contract.
- Calculate the final contract price on a Eurodollar futures contract.
- Describe and compute the Eurodollar Futures contract convexity adjustment.
- Explain how Eurodollar futures can be used to extend the LIBOR zero curve.
- Calculate the duration-based hedge ratio and describe a duration-based hedging strategy using interest rate futures.
- Explain the limitations of using a duration-based hedging strategy

Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.

Commonly used day conventions are:

- **Actual/actual:** U.S. Treasury bonds
- **30/360:** U.S. corporate and municipal bonds
- **Actual/360:** U.S. Treasury bills and **money market instruments**

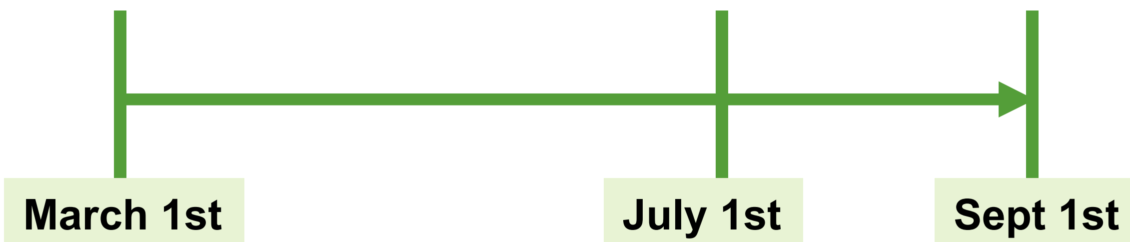
Money Market	Capital Market
• **Short-term financial instruments**	• **Long-term securities**
• **Treasury bill (government)**	• **US Treasury notes (1-10 years) and bonds (> 10 years)**
• **Certificate of deposit (bank)**	• **Domestic and Eurobonds (issued internationally)**
• **Commercial paper (CP)**	• **Euro bond (denominated in Euros)**
• **Repurchase agreement (repo)**	

Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.

Day count conventions are used for computing accrued interest. The interest earned between two dates can be expressed as:

$$\frac{\textbf{No. of days between dates}}{\textbf{No. of days in reference period}} \times (\textbf{Interest earned in the reference period})$$

For example, a bond pays a coupon of $4.00 every six months. If coupons are paid on March 1st and September 1st, and we wish to calculate the accrued interest earned between March 1 and July 3, the following illustrates three different day count conventions applied to the accrued interest (AI) on this bond:



Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation (continued)

Day Count Conventions

Principal	**$100.00**	
Coupon	**8.00%**	*per annum payable semi-annually*

Start Period	Settle-mend	End Period	Day Count Basis	Days Since	Days Total	Coupon	Accrued Inerest
1-Mar	3-Jul	1-Sep	Actual/Actual	124	184	$4.00	$2.696
1-Mar	3-Jul	1-Sep	30/360	122	180	$4.00	$2.711
1-Mar	3-Jul	1-Sep	Actual/360	124	180	$4.00	$2.756

- **Actual/actual** computes based on actual days between March 1st and July 3rd and actual days between March 1st and September 1st: 124 /184 * $4.00 = $2.696
- **30/360** assumes 30 days per month and 360 days per year. In this case, it is 122 days between March 1st and July 3rd and 180 days between March 1st and September 1st: 122/180 * $4.00 = $2.711
- **Actual/360** computes actual days between Mar 1 and Jul 3 and 180 days between Mar 1 and Sep 1(assuming 30 days per month): 124 /180 * $4.00 = $2.756

BIONIC TURTLE

Calculate the conversion of a discount rate to a price for a U.S. Treasury bill.

A US Treasury bill is a "discount instrument:" the discount rate is expressed as a percentage of the face value.

For Example:

- The face value of a 90-day Treasury bill is $100.00 and its cash price is $98.00
- As the T-bill matures in 90 days, the "discount rate" **as a percentage of face value** is 2.0% $(= 100 - 98)$ per quarter. This translates in to a "discount rate" of 8.0% $[= 360/90 * (100 - 98)]$ per annum.

  In this way 8.0% is the annualized $(2.0\% * 4)$ interest as a percentage of the face $([\$2 * 4]/\$100)$. And **8.00 is the quoted price**, but this is *not* the true yield.

BIONIC TURTLE

Calculate the conversion of a discount rate to a price for a U.S. Treasury bill.

A US Treasury bill is a "discount instrument:" the discount rate is expressed as a percentage of the face value. Consequently, the discount rate is not a true yield.

Money market instruments tend to be quoted using a **discount rate**. In general, the relationship between quoted price and cash price of a Treasury bill (if n is the remaining life of the T bill in days) can be given as follows

$$P_{Quoted} = \frac{360}{n} * (100 - Y_{Cash\,price})$$

$$\frac{P_{Quoted} * n}{360} = (100 - Y_{Cash\,price})$$

$$Y_{Cash\,price} = 100 - P_{Quoted}\frac{n}{360}$$

Calculate the conversion of a discount rate to a price for a U.S. Treasury bill (continued)

The "true yield" for this example is illustrated:

The true yield is 8.16% $\left(= \frac{100*8\%}{98}\right)$. This is the quarterly compounded rate such that we can confirm that $\$98 * (1 + \frac{8.1633\%}{4})^{n*4}$ = $100, for n = 0.25 = 90/360.

Discount Rate for Treasury Bill		
Face Value	$100.00	
Days to matury, n	90	
Denominator basis	360	
Quoted price ("discount"), P	8.00	
Cash price, Y	$98.00	= 100 - 8.00 * 90/360
Quoted price	8.00	= 360/90 * (100 - 98.00)
Interest ($)	$2.00	over 90 days
True rate of interest	2.041%	over 90 days

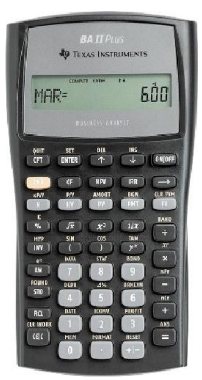
Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond.

Clean (Quoted or Flat) Price

The clean price (aka, *quoted price* or *flat price,* or simply "price") does not reflect the cash price if interest has accrued.

Dirty (Cash or Full) Price

The dirty (aka, full or invoice or cash) price adds accrued interest to the clean price: **Cash Price = (Quoted Price) + (Accrued Interest)**



TVM returns a cash price, not a quoted price

BIONIC TURTLE

Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond (continued)

Cash Price = Quoted Price + Accrued Interest

For example: The quoted price of the 11.0% coupon bond is $95.50 on the settlement date of March 5, 2015 (3/5/2015). 54 days have elapsed since the last coupon and, because we happen to be adopting an actual/actual day count, there are 181 actual days between coupons. The accrued interest is therefore $100.00 * 11.0%/2 * 54/181 = $1.64. The dirty (aka, invoice or full) price of the bond is $95.50 + $1.64 = $97.14.

Hull 6.1: Dirty Price of US Treasury					
				Principal	$100.00
				Coupon	11.0%
Last Coupon	1/10/2015				
Settle Date	**3/5/2015**	**Days Since**	**54**	**Accrued Interest (AI)**	**$1.64**
Next Coupon	7/10/2015	**Total Days**	**181**	**Full Coupon**	**$5.50**
				Clean Price (aka, Quoted or Flat Price)	$95.50
				Dirty Price (aka, Invoice or Full Price)	$97.14

BIONIC TURTLE

Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond (continued)

If P is a bond's flat price, and AI is the accrued interest, then the full price is the present value (PV) which is given by:

$$\mathbf{PV\ (future\ cash\ flows) = P + AI}$$

AI is the accrued interest (we saw it earlier under day count conventions) and is given as:

$$\mathbf{AI = coupon \times \frac{no.\ of\ days\ from\ last\ coupon\ to\ settlement}{no.\ of\ days\ between\ coupons}}$$

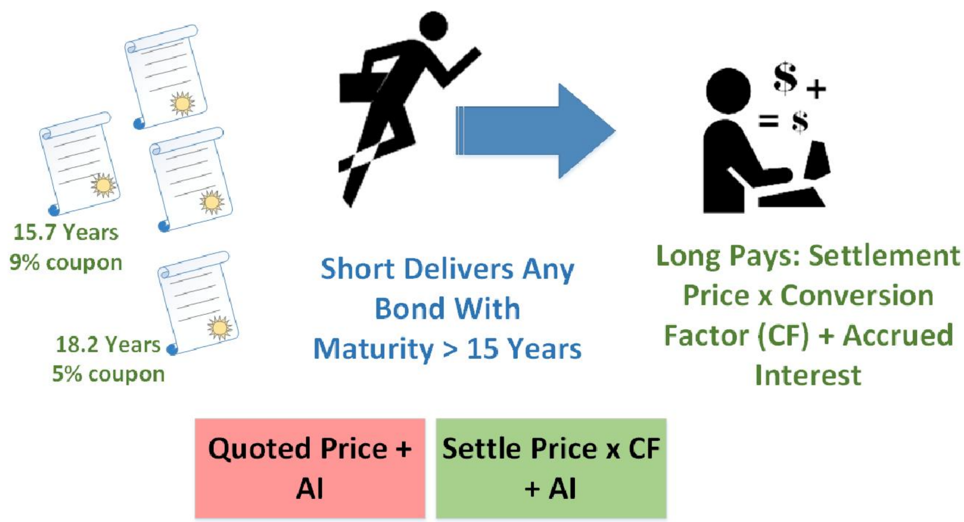
To summarize, the full price of the bond equals the flat price plus accrued interest (if any). The invoice price is the amount paid by the buyer and received by the seller; therefore, it is the face amount multiplied by the full price.

BIONIC TURTLE

Explain and calculate a US Treasury bond futures contract conversion factor.

The Treasury bond futures contract allows the party with the short position to deliver any bond with a maturity of more than 15 years and that is not callable within 15 years. The short here has flexibility in delivery! When the chosen bond is delivered, the **conversion factor(CF)** defines the price received by the party with the short position.

Cash Received by short = Settlement Price × Conversion factor + Accrued interest



Explain and calculate a US Treasury bond futures contract conversion factor (continued)

Conversion factor

The conversion factor for a bond is set equal to the quoted price of the bond per dollar of principal on the first day of the delivery month on the **assumption that the interest rate for all maturities equals 6% per annum (with semiannual compounding)**. The bond maturity and the times to the coupon payment dates are rounded down to the nearest 3 months for the purposes of the calculation.

- If, after rounding, the bond lasts for an exact number of 6-month periods, the first coupon is assumed to be paid in 6 months.
- If, after rounding, the bond does not last for an exact number of 6-month periods (i.e., there are an extra 3 months), the **first coupon is assumed to be paid after 3 months and accrued interest is subtracted.**

Explain and calculate a US Treasury bond futures contract conversion factor (continued)

For Example (Hull 6.2):

Calculation of Conversion Factor

1. Consider a 10% coupon bond with $100 face value, and 20 years and 2 months to maturity.
2. Consider another bond with a coupon of 8% with 18 years and 4 months to maturity

For calculation of conversion factor, a discount rate of 6% per annum with semiannual compounding (or 3% per 6 months) is assumed. For the purposes of calculating the conversion factor, the first bond is assumed to have exactly 20 years to maturity while the second bond is assumed to have exactly 18 years and 3 months to maturity.

Explain and calculate a US Treasury bond futures contract conversion factor (continued)

- The value of the first bond is the present value of all its cash flows and is found to be $146.23. The conversion factor is the quoted price per dollar and is got by dividing this bond price by the face value to give 1.4623 (=146.23/100)
- For the second bond, the first coupon is assumed to be paid after 3 months and the accrued interest is subtracted to obtain the price of the bond, which is $123.99. So, the conversion factor is 1.2199 (= [123.99 – 2]/100)

	Bond #1	Bond #2
Face value	$100.00	$100.00
Coupon	10.00%	8.00%
Yield	6.00%	6.00%
Maturity (years)	20.17	18.33
Round down, nearest 3 mos	20.00	18.25
In six or three months?	six	three
[if three] PV + 0.25	na	$125.832
[if three] discounted	na	$123.986
[if three] minus AI	na	$121.986
Price	$146.430	$121.986
Converstion Factor, CF	1.4643	1.2199

BIONIC TURTLE

Calculate the cost of delivering a bond into a Treasury bond Futures contract.

During the delivery month, many bonds (with varying coupon and maturity) can be delivered in the Treasury bond futures contract. The party with the short position can choose which of the available bonds is ‘‘cheapest’’ to deliver.

The cost to deliver by the short position is the dirty price, which is the quoted bond price plus accrued interest (AI).

The short position will receive the settlement price times conversion factor plus accrued interest (AI) from the long.

The **cheapest to deliver (CTD)** bond is the one which minimizes the difference between the cost paid and the cash received by the short. In other words, CTD bond is the bond that either:

minimizes → MIN: Quoted Bond Price - (Settlement Price × CF) or

maximizes → MAX: (Settlement Price × CF) - Quoted Bond Price

BIONIC TURTLE

Calculate the cost of delivering a bond into a Treasury bond Futures contract (continued)

Hull Example 6.1

The party with the short position has decided to deliver and is trying to choose between the three bonds in the table below.

- The most recent settlement price is 93-08 or 93.25.
- For example, the cost to deliver the second bond is 143.5 – (93.25 × 1.5188) = $1.87.

Cost of delivering a Treasury bond and CTD bond

Short receives:	(Settlement)(CF) + AI
Cost ("dirty price"):	Quoted Bond Price + AI
CTD Minimizes	Quoted Bond Price - (Settlement)(CF)
Settlement price	**$93.25**

Bond	Quoted Price	CF	Cost
1	$99.50	**1.0382**	$2.69
2	$143.50	**1.5188**	$1.87
3	$119.75	**1.2615**	$2.12
Bond 2 is CTD because its cost is:			**$1.87**

#2 is cheapest to deliver (CTD)

BIONIC TURTLE

Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision.

What determines the ***cheapest-to-deliver (CTD***) bond?

Are yields above or below the 6.0% level?

What is the slope of the yield curve.

Yield levels: Because the CTD is based on standardizing the yields at 6% ...

- ❑ If yields are **below 6.0%**, CTD systems favors delivery of high-coupon, short-maturity bonds; i.e., ***bonds with lower durations***
- ❑ If yields are **above 6.0%**, CTD systems favors delivery of low-coupon, long-maturity bonds; i.e., ***bonds with higher durations***.

Slope of yield curve:

- ❑ An **upward-sloping** yield curve favors long time-to-maturity bonds
- ❑ A **downward-sloping** yield curve favors short time-to-maturity bonds.

Calculate the theoretical futures price for a Treasury bond futures contract.

The futures contract on a Treasury bond provides the holder with known income assuming that both the cheapest-to-deliver bond and the delivery date are known. Its futures price is given by:

$$F_0 = (S_0 - I)e^{rT}$$

Hull Example 6.2:

Cheapest to deliver (CTD) bond is a 12.0% coupon bond with a conversion factor of 1.60 and delivery in 270 days. Coupons pay $6.00 semi-annually and the last coupon was paid 60 days ago, the next coupon pays in 122 days, and the coupon thereafter is in 305 days. The term-structure is flat with interest rate at 10% per annum. Assume current quoted bond price is $115.00.

Hull Example 6.2: Theoretical Price of Treasury Bond Futures Contract
Assumes cheapest to deliver (CTD) will be a 12.0% coupon with CF 1.60

Assumptions	
Face	$100.00
Current Quoted (Clean) Price	$115.00
Coupon	12.0%
Interest rate	10.0%
Conversion Factor	1.60
Delivery (days)	270
Last Coupon (-days)	60
Next Coupon (+ days)	122
Coupon thereafter	305

Timeline	
	Coupon payment
-60 days	
	Current time
122 days	
	(Next) coupon payment
148 days	
	Maturity of futures
35 days	
	Coupon payment

Solution (Hull's Example 6-2)	
Accrued Interest	$1.978
Cash (Dirty, Full) Price	$116.978
PV of next coupon	$5.803
Cash Futures Price	$119.711
Days Accrued @ delivery	148
Days Remain @ delivery	35
Futures price, Quoted, 12% bond	$114.859
Futures price, Quoted, CTD	$71.787

COC: $F_0=(S_0-I)$*EXP(rT)	
1. COC to calculate Cash forward	
Spot:	$116.98
Income:	$5.803
Forward cash price:	$119.711
2. Then dirty --> clean & standardize	
Subtract AI	$4.852
Quoted Price, 12% bond	$114.859
Divide by CF of 1.60	$71.787

Calculate the theoretical futures price for a Treasury bond futures contract (continued)

- The ***cash price*** is quoted bond price plus the proportion of the next coupon payment that accrues to the holder.
  Cash price = Quoted bond price + AI

$$\textbf{Cash price} = \mathbf{115} + \left(\frac{\mathbf{60}}{\mathbf{60} + \mathbf{122}} \times \mathbf{6}\right) = \mathbf{115} + \mathbf{1.978} = \$\mathbf{116.978}$$

- ***PV of $6 coupon*** to be received after 122 days is:

$$\textbf{PV of coupon} = \mathbf{6}e^{\mathbf{10\%}\times(\mathbf{122/36}} = \mathbf{5.803}$$

- ***Cash futures price***, if the contract were written on the 12% bond, for delivery in 270 days is:

$$\textbf{Cash futures price} = (\mathbf{116.978}\ \mathbf{5.803}) \times e^{\mathbf{10\%} \times (\mathbf{270/365})} = \$\mathbf{119.711}$$

BIONIC TURTLE

Calculate the theoretical futures price for a Treasury bond futures contract (continued)

- At delivery, there are 148 (=270 -122) days of accrued interest. The days remaining at delivery is 35(=305 - 270). The ***quoted futures price*** for the 12% bond is obtained by subtracting this AI from the cash futures price.

$$\textbf{Quoted futures price} = \mathbf{119.711} - \left(\frac{\mathbf{148}}{\mathbf{148+35}} \times \mathbf{6}\right) = \mathbf{\$114.859}$$

- From the conversion factor, 1.6 standard bonds are considered equivalent to each 12% bond. The ***quoted futures price*** should therefore be

$$\textbf{Quoted Futures price (CTD)} = \mathbf{114.859 / 1.6 = 71.79}$$

Also, the ***cash futures price*** is based on the cost of carry model as:

$$F_0 = (S_0 - I)e^{rT} = (116.978 - 5.803) \times e^{10\% \times (270/365)} = \$119.711$$

From this dirty price, we subtract the AI and standardize (divide by CF) to get the ***clean or quoted futures price***: $\dfrac{119.711 - \left(\frac{148}{148+35} \times 6\right)}{1.6} = 71.79$

Calculate the theoretical futures price for a Treasury bond futures contract (continued)

Hull Example 6.2: Theoretical Price of Treasury Bond Futures Contract
Assumes cheapest to deliver (CTD) will be a 12.0% coupon with CF 1.60

Assumptions	
Face	**$100.00**
Current Quoted (Clean) Price	**$115.00**
Coupon	**12.0%**
Interest rate	**10.0%**
Conversion Factor	**1.60**
Delivery (days)	**270**
Last Coupon (-days)	**60**
Next Coupon (+ days)	**122**
Coupon thereafter	**305**

Timeline	
	Coupon payment
-60 days	
	Current time
122 days	
	(Next) coupon payment
148 days	
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COC: $F_0=(S_0-I)*EXP(rT)$	
1. COC to calculate Cash forward	
Spot:	**$116.98**
Income:	**$5.803**
Forward cash price:	**$119.711**
2. Then dirty --> clean & standardize	
Subtract AI	**$4.852**
Quoted Price, 12% bond	**$114.859**
Divide by CF of 1.60	**$71.787**

BIONIC TURTLE

Calculate the final contract price on a Eurodollar futures contract.

If R is the LIBOR interest rate, the Eurodollar futures price is quoted at 100–R.

The contract is designed so that a one-basis-point (.01%) move in the futures quote corresponds to a gain or loss of $25 per contract. A one-basis-point change in the futures quote corresponds to a 0.01% change in the underlying interest rate such that:

$$1{,}000{,}000 \times 0.0001 \times 0.25 = 25$$

The contract price is defined as: **10,000 × [100 – 0.25 × (100 – Quote)]**

If the Eurodollar Futures quote increases by one (1) basis point, long position gains $25 and short position loses $25.

If the Eurodollar Futures quote decreases by one (1) basis point, long position loses $25 and short position gains $25

Calculate the final contract price on a Eurodollar futures contract.

If R is the LIBOR interest rate, the Eurodollar futures price is quoted at 100–R.

The contract price is defined as: 10,000 × [100 – 0.25 × (100 – Quote)]

For example: In May 2013, an investor buys a Eurodollar contract at quote of 99.725 (implied LIBOR = 100 – 99.725 = 0.275%). Going forward to June, when contract settles, LIBOR is 0.385%, so quote is 99.615 (100 – 0.385 = 99.615). The difference between the quotes is 0.11(99.725 - 99.615).

- May contract price = 10,000 × [100 – 0.25 × (100 – 99.725)] = $999,312.5
- June (settlement) contract price = 10,000 × [100 – 0.25 × (100 – 99.615)] = $990,037.5

Calculate the final contract price on a Eurodollar futures contract (continued)

Hull Table 6.2: Eurodollar Futures Contract
By design, a one-basis-point move in quote --> +/- $25.00

Date	Trade Price	Settlement Futures Price	Price Δ	Price Δ (bps)	Contract Price	Gain per long contract
May 3, 2016	99.330				$998,325.00	
May 3, 2016		99.325	-0.005	(0.5)	$998,312.50	-$12.50
May 4, 2016		99.275	-0.050	(5.0)	$998,187.50	-$125.00
		...				
June 13, 2016		99.220	0.010	1.0	$998,050.00	$25.00
		Total:	-0.110	(11.0)		-$275.00

BIONIC TURTLE

Describe and compute the Eurodollar Futures contract convexity adjustment.

The convexity adjustment assumes continuous compounding.

- Given that (σ) is the standard deviation of the change in the short-term interest rate in one year, T_1 is the time to maturity of the futures contract and T_2 is the time to maturity of the rate underlying the futures contract, convexity adjustment is: $\frac{1}{2}\sigma^2 T_1 T_2$
- Under the Hull-Lee model, the forward rate is less than the futures rate as a function of variance:

$$\boldsymbol{Forward\ rate = Futures\ rate - \frac{1}{2}\sigma^2 T_1 T_2}$$

The primary difference is due to daily settlement of futures contracts: as they settle daily, it leads to interim cash flows (i.e., margin calls or excess margin).

BIONIC TURTLE

Describe and compute the Eurodollar Futures contract convexity adjustment (continued)

Hull Example 6.4

Consider the assumptions given in the table below for an 8-year Eurodollar futures whose price quote is 94.

- The convexity adjustment is: $\frac{1}{2}\sigma^2 T_1 T_2$ =0.475%
- The futures rate is 6.00% per annum on an actual/360 basis with quarterly compounding.
- This corresponds to 1.50% per quarter or an annual rate 6.038% with continuous compounding and an actual/365 day count.
- The forward rate after adjusting the futures rate for convexity is 5.563% per annum with continuous compounding.

Convexity Adjustment

Volatility of short rate	1.20%	
Eurodollar Futures price	94	
T1	8	
T2 (three month rate)	8.25	
Convexity adjustment	0.475%	= 0.5 * 1.2%^2 * 8 * 8.25
Futures rate (ACT/360)	6.00%	= 100 - 94 price
	1.50%	Per 90 days
Futures rate (Cont. & Act/360)	6.038%	= LN (1.015)*365/90
Forward (continuous)	5.563%	= futures - convexity adj

BIONIC TURTLE

Explain how Eurodollar futures can be used to extend the LIBOR zero curve.

The bootstrap procedure can be used to extend the LIBOR zero curve. We know that the forward interest rate F_i is:

$$F_i = \frac{R_{i+1}T_{i+1} - R_iT_i}{T_{i+1} - T_i}$$

From this forward rate, the zero rate can be obtained as:

$$\boldsymbol{R_{i+1} = \frac{F_i\,(T_{i+1} - T_i) + R_iT_i}{T_{i+1}}.}$$

BIONIC TURTLE

Explain how Eurodollar futures can be used to extend the LIBOR zero curve (continued)

Hull Example 6.5

The 400-day LIBOR zero rate has been calculated as 4.80% with continuous compounding and, from Eurodollar futures quotes, the forward rate for a 90-day period beginning in 400 days, 491 days and 589 days has been calculated as 5.30%, 5.50% and 5.60% respectively, with continuous compounding.

- The 491-day zero rate is obtained using the forward rate of 5.3% as:

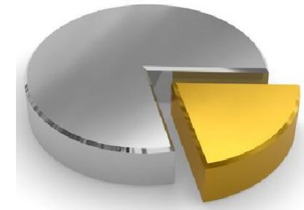
$$\frac{5.3\% \times (491 - 400) + 4.8\% \times 400}{491} = 4.893\%$$

- Similarly, the 589-day zero rate of 4.994% is obtained using 5.5% forward rate

Starting in (days)	Period (days)	Forward Rate	Zero Rate
0	400	4.80%	
400	90	5.30%	4.800%
491	90	5.50%	4.893%
589	90	5.60%	4.994%

BIONIC TURTLE

Calculate the duration-based hedge ratio and describe a duration-based hedging strategy using interest rate futures.



The number of contracts required to hedge against an uncertain change in the yield(Δy) is the duration-based hedge ratio and is given by:

$$N^* = \frac{PD_P}{V_F D_F}$$

V_F : contract price for the interest rate futures contract.

D_F: duration of asset underlying futures contract at maturity.

P : forward value of the portfolio being hedged at the maturity of the hedge (typically assumed to be today's portfolio value).

D_P: duration of portfolio at maturity of the hedge.

BIONIC TURTLE

Calculate the duration-based hedge ratio and describe a duration-based hedging strategy using interest rate futures (continued)

Hull Example 6.6

Assume a portfolio value of $10 million invested in government bonds. The manager hedges with T-bond futures (each contract delivers $100,000) with a current price of $93.0625. The duration of the portfolio at hedge maturity will be 6.8 and the duration of futures contract will be 9.2. How many futures contracts should be shorted?

$$N^* = \frac{PD_P}{V_F D_F} = \frac{(10{,}000{,}000)(6.8)}{(93{,}062.5)(9.2)} = 79.4\ 2$$

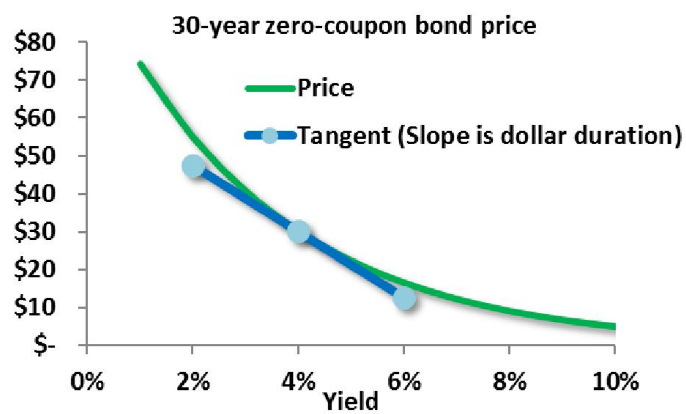
So, the manager should short 79 contracts to hedge the bond portfolio.

BIONIC TURTLE

Calculate the duration-based hedge ratio and describe a duration-based hedging strategy using interest rate futures (continued)

Duration-based hedging strategy using interest rate futures

- **Duration matching or portfolio immunization:** Financial institutions may hedge themselves against interest rate risk by ensuring that the average duration of their assets equals the average duration of their liabilities. The liabilities can be regarded as short positions in bonds.
- When implemented, it ensures that a **small parallel shift** in interest rates will have little effect on the value of the portfolio of assets and liabilities. The gain (loss) on the assets should offset the loss (gain) on the liabilities.



Explain the limitations of using a duration-based hedging strategy

Although matching the durations of assets and liabilities is sometimes a first step in ***asset–liability management (ALM)*** to monitor exposure to interest rates, this does not protect a bank against non-parallel shifts in the yield curve. This is a weakness of the duration based hedging approach.

- In practice, short-term rates are usually more volatile than, and are not perfectly correlated with, long-term rates.
- Sometimes it even happens that short- and long-term rates move in opposite directions to each other.

Therefore, additional tools are required to manage interest rate exposure. Tools such as swaps, FRAs, bond futures, Eurodollar futures, and other interest rate derivatives can be used for this purpose.

The End

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Interest Rate Futures