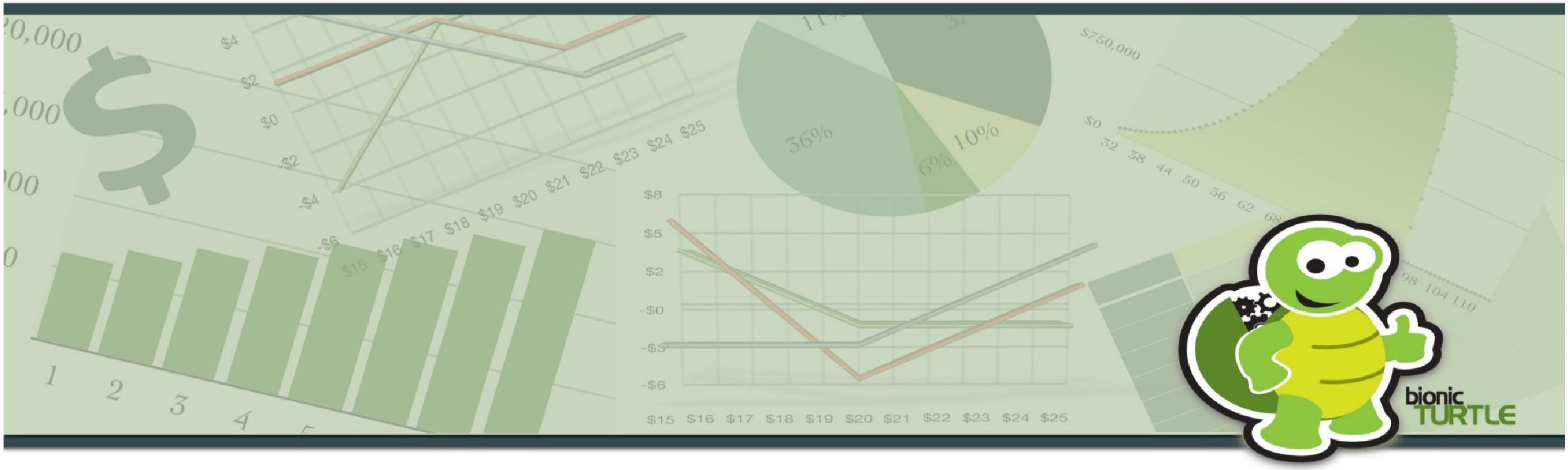


# P1.T3. Financial Markets & Products

## Hull, Options, Futures & Other Derivatives
## Swaps

# Bionic Turtle FRM Video Tutorials

By David Harper, CFA FRM

# Swaps: Conceptual LOs

**Key ideas:**

1. **Swaps to transform an asset or liability (fixed to floating, or vice-versa)**
2. **Comparative advantage**
3. **Credit exposure = Max(0, MTM)**
4. **Other swaps**

- Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.
- Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.
- Explain the role of financial intermediaries in the swaps market.
- Describe the role of the confirmation in a swap transaction.
- Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument.

….

- Describe the credit risk exposure in a swap position.
- Identify and describe other types of swaps, including commodity, volatility and exotic swaps.

## Swaps: Quantitative LOs

…

- Explain how the discount rates in a plain vanilla interest rate swap are computed.
- Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.

**Key ideas:**

1. **What is a swap rate?**
2. **In Hull 10th Edition, OIS zero rate is the discount rate!**
3. **IRS valuation: 2 bonds or FRAs**
4. **Currency swap valuation: 2 bonds or FRAs**

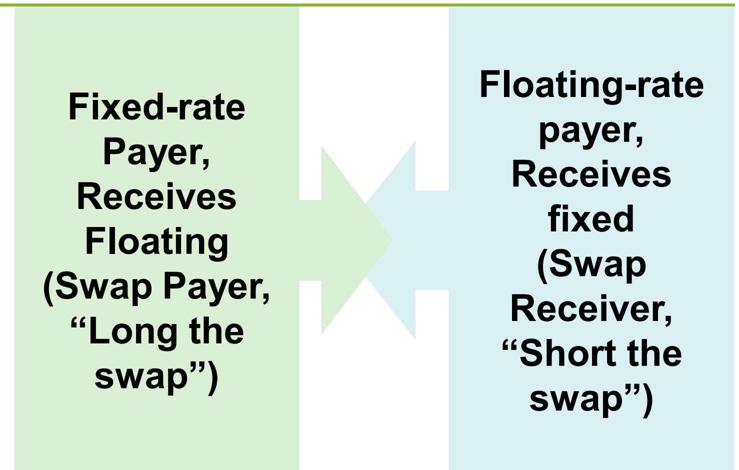
- Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).
- Explain the mechanics of a currency swap and compute its cash flows.
- Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows.
- Calculate the value of a currency swap based on two simultaneous bond positions.
- Calculate the value of a currency swap based on a sequence of FRAs.

…

# Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.

In a ***plain-vanilla interest-rate swap***, one company agrees to

- **Pay a predetermined fixed interest rate** in exchange for …
- **Receiving a variable interest rate;** i.e., *pay-fixed and receive-floating*
- Their counterparty agrees to pay a variable rate and receive a fixed interest rate (*pay-floating and receive-fixed*).




The counterparty **who pays the fixed rate is called the swap payer.** This counterparty profits from an ***increase*** in interest rates

The counterparty **who pays the floating rate is called the swap receiver**. This counterparty profits from a ***decrease*** in interest rates

# Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows (continued)

| | |
|---|---|
| Principal | $100.00 |
| Fixed rate | 3.0% |

| Date | End of Period (6 months) | LIBOR at Start of Period | Cash Flows Pay Fixed | Cash Flows Receive Floating | Net Cash Flow |
|---|---|---|---|---|---|
| Mar-8-2017 | | 2.20% | | | |
| Sep-8-2017 | 1 | 2.80% | (1.50) | +1.10 | (0.40) |
| Mar-8-2018 | 2 (Yr 1) | 3.30% | (1.50) | +1.40 | (0.10) |
| Sep-8-2018 | 3 | 3.50% | (1.50) | +1.65 | 0.15 |
| Mar-8-2019 | 4 (Yr 2) | 3.60% | (1.50) | +1.75 | 0.25 |
| Sep-8-2019 | 5 | 3.90% | (1.50) | +1.80 | 0.30 |
| Mar-8-2020 | 6 (Yr 3) | | (1.50) | 1.95 | 0.45 |

**Floating rate determined at beginning of period, paid at end of period**

**Notional not exchanged!**

Illustration assumes:

- ***Notional principal:*** $100 million. It is called *notional* principal: in the "vanilla" swap **principal is not exchanged**.
- ***Swap agreement:*** Pay fixed rate of 3.0% per annum, and receive 6 month LIBOR rate
- ***Term (aka, tenor):*** 3 years with payments every six months

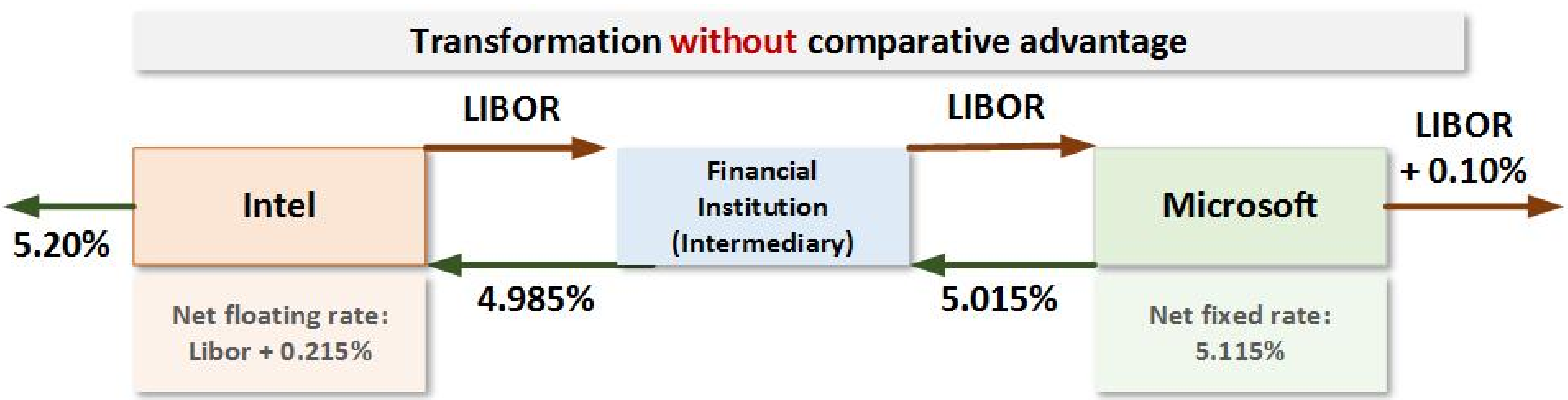
BIONIC TURTLE

# Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.

A company could use a swap to transform a floating-rate loan into a fixed-rate loan.

Suppose Microsoft (MSFT) can borrow $100.0 million *in the floating-rate market* at LIBOR plus 10 basis points. Assume Microsoft **prefers to borrow instead at fixed rates.** Microsoft enters the swap such that:

- **MSFT pays LIBOR plus 0.10% to its lenders.**
- **Under the terms of the swap, MSFT pays 5.015% fixed**
- **Under the terms of the swap, MSFT receives LIBOR**



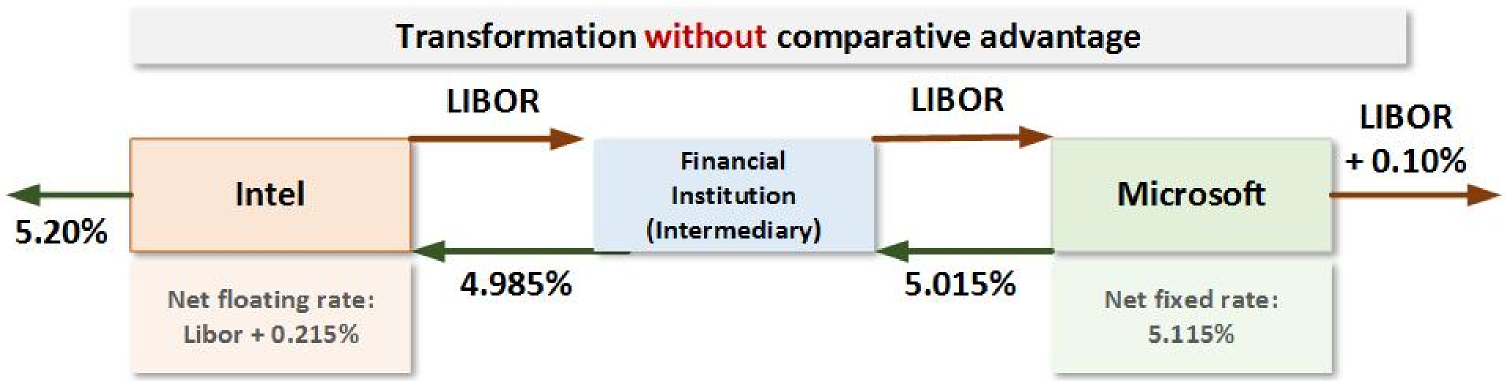

BIONIC TURTLE

# Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows (continued)

The swap effectively transforms borrowings at a floating rate of LIBOR plus 0.1% into borrowings at a **net fixed rate of 5.115%**; i.e., LIBOR cancels and MSFT pays 5.015% plus 10 basis points. Suppose Intel (INTC) is the swap counterparty. Intel borrows 5.20 in fixed rate markets but prefers to pay a floating rate. After Intel enters into the swap:

- **Intel pays 5.20% to its lenders.**
- **Intel pays LIBOR under the terms of the swap.**
- **Intel receives 4.985% under the terms of the swap.**

This transforms Intel's borrowing to a net floating rate of LIBOR + 215 basis point.




BIONIC TURTLE

# Explain the role of financial intermediaries in the swaps market.

A financial intermediary may earn about 3 or 4 basis points (0.03% or 0.04%) on a pair of offsetting transactions of a "plain vanilla" LIBOR-for-fixed swaps. The spread earned is partly to compensate it for the risk that one of the two companies will default on the swap payments.

Here we discuss the potential uses of a swap for the same two companies mentioned in the previous slides. A financial intermediary(FI) enters into two offsetting swap transactions with both Intel and Microsoft.

- Assuming that both companies honor their obligations, the financial institution is certain to make a profit of $30,000 per year (0.03% times notional principal of $100 million).
- Microsoft ends up borrowing at 5.115% (instead of 5.1%), and Intel ends up borrowing at LIBOR plus 21.5 basis points (instead of at LIBOR plus 20 basis points) if they transact through an intermediary instead of directly entering into a swap with each other as explained in the previous section.

# Describe the role of the confirmation in a swap transaction.


**Confirmation is a legal agreement underlying a swap and is signed by representatives of the two parties. Drafting of confirmations is facilitated by the ISDA.**

- ISDA has produced a number of master agreements and the so-called credit support annexes that include well-defined clauses.
- In the US, it is commonplace for companies to require an ISDA master agreement be entered into before entering into a swap transaction. This is then supplemented with a credit support annex, which stipulates further terms specific to the transaction.
- An ISDA agreement can include description of the terms of the swap contracts including netting arrangements, collateral and non-performance clauses. It would be highly unusual for a firm to enter into a swap transaction in the OTC market without some form of agreement with the ISDA.

BIONIC TURTLE

# Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument.

**The comparative-advantage argument is used to explain the popularity (or utility) of swaps.** Consider two companies: AAACorp has a higher credit rating than BBBCorp. Their respective borrowing rates are:

Absolute advantage in both markets

| | Fixed | Floating |
|---|---|---|
| AAACorp | 4.0% | 6 mo. LIBOR – 0.1% |
| BBBCorp | 5.2% | 6 mo. LIBOR + 0.6% |

- **AAACorp has an advantage in *both* the fixed and floating markets.** When one company has an advantage in a market, it is called an ***absolute advantage***.
  - Despite the *absolute* advantage of AAACorp in both markets, the fact that BBBCorp enjoys a ***comparative advantage*** in floating-rate markets implies they can achieve mutual gain!

# Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument.

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| | Fixed | Floating |
|---|---|---|
| AAACorp | 4.0% | 6 mo. LIBOR – 0.1% |
| BBBCorp | 5.2% | 6 mo. LIBOR + 0.6% |

**Comparative advantage in floating-rate market!**

- **BBBCorp is said to have a *comparative advantage* in the floating-rate market** (because BBBCorp borrows at only +0.70% more in floating rate markets, compared to 1.2% more in fixed rate markets).
  - AAACorp is said to have a *comparative advantage* in fixed rate markets.

BIONIC TURTLE

## Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument (continued)

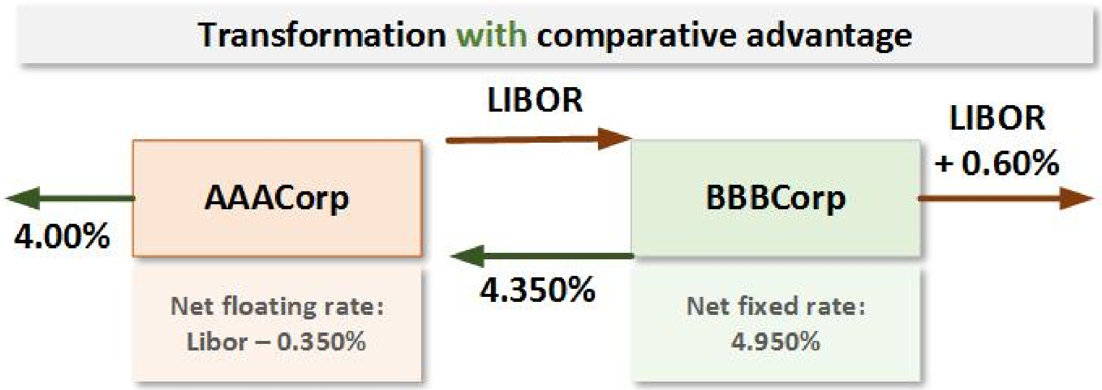
| | Fixed | Floating | |
|---|---|---|---|
| AAACorp | 4.0% | LIBOR – 0.1% | |
| BBBCorp | 5.2% | LIBOR + 0.6% | |
| | 1.2% | 0.70% | +0.50 |

- The total advantage is given by the **difference between the respective rate differentials**. Specifically, in this case, the fixed rate differential equals 1.20% = 5.20% - 4.00%, and the floating-rate differential equals 0.70% = 0.60% -(-) 0.10%. The total gain equals 0.50% = 1.20% - 0.70%.
- To generalize, we can say that a *comparative advantage* exists when two companies face different interest rate markets: the difference in fixed rate markets (i.e., between the companies; call this "$a$") is greater than the difference in floating rate markets (call this "$b$"). Under these circumstances, **a swap arrangement can produce a total gain, that is, to both parties, before any transaction costs, equal to: $a - b$.**

BIONIC TURTLE

# Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument (continued)

If AAACorp and BBBCorp want to share the advantage equally, they swap as follows:




In this arrangement, AAACorp's cash flows (ignoring transaction costs) are:

- It pays 4.00% per annum to outside lenders.
- It receives 4.35% per annum from BBBCorp.
- It pays LIBOR to BBBCorp.

BBBCorp's cash flows are:

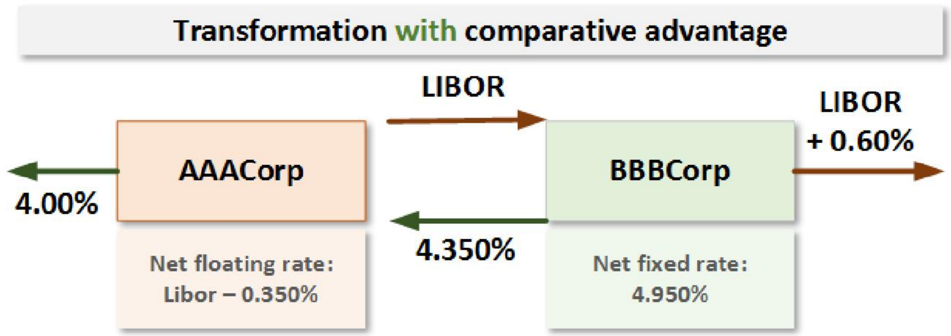
- It pays LIBOR + 0.6% per annum to outside lenders.
- It receives LIBOR from AAACorp.
- It pays 4.35 % per annum to AAACorp.

BIONIC TURTLE

# Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument (continued)

Under this swap, both have improved
their cost of capital as they effectively pay:

- **AAACorp pays LIBOR - 0.35%: 0.25% less than its own floating rate,**
- **BBBCorp pays 4.95% fixed: 0.25% less than its own fixed rate**

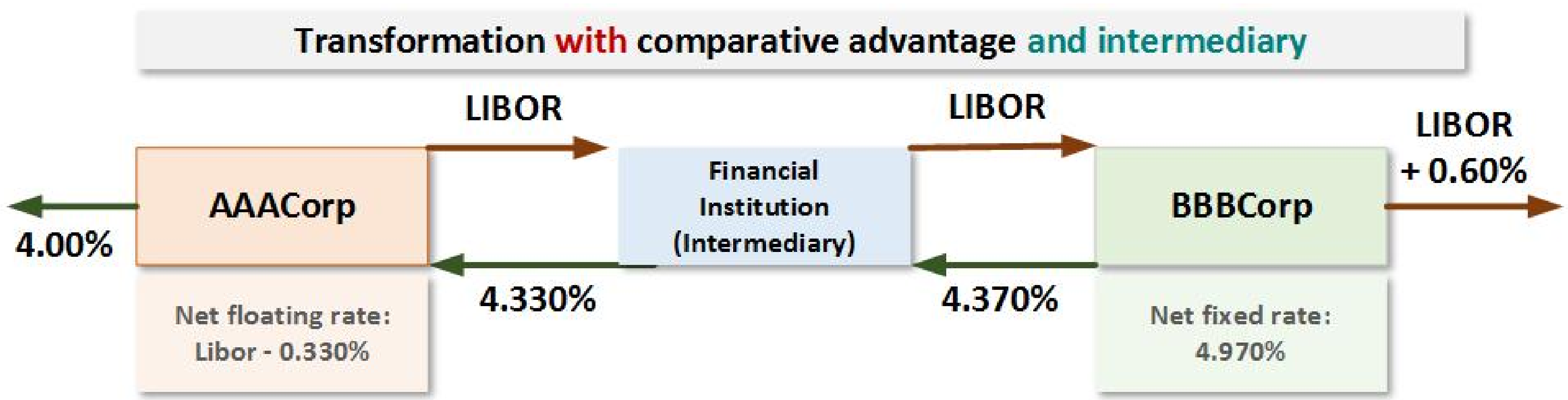



In this swap arrangement, the total gain $a - b$ should be 1.2- 0.7 = 0.5%. As we have seen above, the net gain of 0.25% on each side sums up so that the total gain is 0.5%.

# Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument (continued)

**Inserting a financial intermediary into the swap**

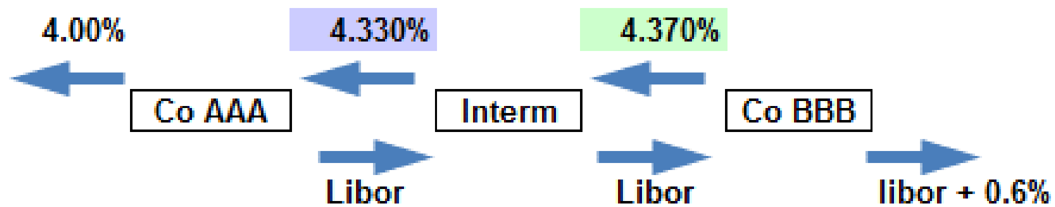
We can add an assumption that the two counterparties do not deal directly with each other but instead use a financial intermediary. In this case, we continue to follow Hull's example and assume the intermediary charges four basis point (0.04%). The reduces the total shared net advantage from 50 basis points to 46 basis points.



# Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument (continued)

| | Fixed | Float: libor + | |
|---|---|---|---|
| AAACorp | 4.00% | -0.10% | fixed advantage, but SEEKS to borrow floating |
| BBBCorp | 5.20% | 0.60% | floating advantage, but SEEKS to borrow fixed |
| | 1.20% | 0.70% | |

| | | |
|---|---|---|
| 1.20% - 0.70% = | 0.50% | ← Gross total advantage |
| | 0.04% | ← Intermediary |
| | 0.46% | ← Net total advantage |




| | | | |
|---|---|---|---|
| AAACorp | 50% | | of advantage |
| | 0.230% | | Gain to Company A |
| | 4.000% | | External (original) borrowing, where it has advantage |
| | | -0.330% | = Libor + (-0.10% - 0.23%) is NET borrowing |
| | | 4.330% | In swap, pay Libor in exchange for this fixed rate |
| BBBCorp | 50% | | of advantage |
| | 0.230% | | Gain to Company B |
| | | 0.600% | L+x, External borrowing, where it has advantage |
| | 4.970% | | = 5.20% - 0.23% is NET borrowing |
| | | 4.370% | SWAP: receive Libor, pay this fixed rate |
| | | 4.370% | should be same as this |

# Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument (continued)


**Criticism of the comparative advantage argument**

**The contrary view concerns arbitrage: if markets were efficient, we would expect the differentials that allow for the comparative advantage in the first place, to erode.**
The reason that spread differentials appear to exist is due to the nature of the contracts available to companies in fixed and floating markets.

- The floating rate is typically LIBOR + a spread, and is adjusted, or *reset*, every six months. Thus, if the borrower's creditworthiness has declined, the lender has the option of increasing the spread over LIBOR that is charged. This is not possible for fixed rate loans as they are set for a longer period of time.
- In the short term, there is very little chance that either of the firms will default but as we look further ahead, the probability of a default by a company with a relatively low credit rating is liable to increase faster than the probability of a default by a company with a relatively high credit rating. So, the spread between the 5-year rates is greater than the spread between the 6-month rates.

BIONIC TURTLE

## Explain how the discount rates in a plain vanilla interest rate swap are computed.

**LIBOR rates are observable only for short time periods, typically from one day to one year.**

- For longer durations, typically 1 year to 3 years, Eurodollar futures are used, and then from year 3 to year 30, the “swap curve” is used.
- The reason multiple curves are used has to do with the liquidity of the instrument at the different time-horizons.
- To compute the discount rate for the LIBOR/swap zero curve, we can use the bootstrap method.

# Explain how the discount rates in a plain vanilla interest rate swap are computed (continued)

**For example:**

- Assume that LIBOR/swap zero rates are given: six-month = 4%, one-year = 4.5%, and eighteen months = 4.8%.
- The 2-year swap rate is 5.0%, which implies that a $100.00 face value bond with a semiannual coupon of 5% will sell exactly at par (why? Because the 5% coupons are discounted at 5%)

We can solve for the two year zero rate (R) which is the unknown as follows:

$$2.5e^{(-4\%*0.5)} + 2.5e^{(-4.5\%*1)} + 2.5e^{(-4.8\%*1.5)} + 102.5e^{(-R*2)} = 100$$

$$2.45 + 2.39 + 2.33 + 102.5e^{(-R*2)} = 100 \quad \rightarrow \quad 102.5e^{(-R*2)} = 92.83$$

$$e^{(-R*2)} = 92.83/102.5 \quad \rightarrow \quad R = -\ln(92.83/102.5)/2 = 4.953\%$$

BIONIC TURTLE

# Explain how the discount rates in a plain vanilla interest rate swap are computed (continued)

**Hull 9th Edition**
**Bootstrapping**
**2-year zero rate**

**For example:**

- Assume that LIBOR/swap zero rates are given: six-month = 4%, one-year = 4.5%, and eighteen months = 4.8%.
- The 2-year swap rate is 5.0%, which implies that a \$100.00 face value bond with a semiannual coupon of 5% will sell exactly at par (why? Because the 5% coupons are discounted at 5%)

| | |
|---|---|
| Par | \$100.00 |
| 2-year swap rate | 5.00% |

| Period | Future Cash flow | LIBOR/ swap zero rates (CC) | Present Value Cash flow | |
|---|---|---|---|---|
| 0.50 | \$2.50 | 4.00% | \$2.45 | |
| 1.00 | \$2.50 | 4.50% | \$2.39 | |
| 1.50 | \$2.50 | 4.80% | \$2.33 | |
| 2.00 | \$102.50 | 4.953% | \$92.83 | = \$100.00 -\$2.45 -\$2.39 -\$2.33 |
| | | Sum (PV) | \$100.00 | |
| | | 4.953% | = -LN(\$92.83/\$102.50)/2 | |

$$2.5e^{(-4\%*0.5)} + 2.5e^{(-4.5\%*1)} + 2.5e^{(-4.8\%*1.5)} + 102.5e^{(-R*2)} = 100$$

# Explain how the discount rates in a plain vanilla interest rate swap are computed (continued)

**Hull 10th Edition: Bootstrapping 2-year forward rate**

**Hull's 10th Edition: Discounting with OIS zero rates**

- Assume continuous OIS zero rates are 3.80%, 4.30%, 4.60%, and 4.75%
- Six-month semi-annual LIBOR = 4% and forward LIBOR rates are F(0.5, 1.0) = 5.0% and F(1.0, 1.5) = 5.5%
- We can solve (bootstrap) for the LIBOR forward, F(1.5, 2.0):

| | |
|---|---|
| Par | $100.00 |
| 2-year swap rate | 5.00% |

| Period | OIS zero rates (CC) | Forward LIBOR (s.a.) | Cash Flow FV | Cash Flow PV | |
|---|---|---|---|---|---|
| 0.50 | 3.80% | 4.000% | ($0.500) | ($0.491) | |
| 1.00 | 4.30% | 5.000% | $0.000 | $0.000 | |
| 1.50 | 4.60% | 5.500% | $0.250 | $0.233 | |
| 2.00 | 4.75% | 5.566% | $0.283 | $0.257 | = $0.00 --$0.49 -$0.00 -$0.23 |
| | | | | $0.00 | |

$$0.5 \times (0.04 - 0.05) \times 100 \times e^{-0.0380 \times 0.5} = -0.4\ 9\ 06$$

$$0.5 \times (0.05 - 0.05) \times 100 \times e^{-0.04\ 30 \times 1.0} = 0$$

$$0.5 \times (0.055 - 0.05) \times 100 \times e^{-0.04\ 60 \times 1.5} = 0.2333$$

$$0.5 \times (F - 0.05) \times 100 \times e^{-0.04\ 75 \times 2} = 0.2573$$

5.566% = $0.283/($100.00*0.5) + 5.00%

BIONIC TURTLE

# Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.

| Here is the notation: | Symbol |
|---|---|
| Time until $i^{th}$ payments are exchanged: | $t_i$ |
| Notional principal in swap agreement: | $L$ |
| Fixed payment made on each payment date: | $k$ |
| The next floating-rate payment to be made on the next payment date: | $k^*$ |

**Interpretation of Swap**

- If two companies enter into an interest rate swap arrangement, then one of the companies has a swap position that is equivalent to a long position in floating-rate bond and a short position in a fixed-rate bond:

$$V_{\text{Swap}} = B_{\text{Float}} - B_{\text{Fixed}}$$

- The counterparty to the same swap has the equivalent of a long position in a fixed-rate bond and a short position in a floating-rate bond:

$$V_{\text{Swap Counterparty}} = B_{\text{Fixed}} - B_{\text{Float}}$$

BIONIC TURTLE

# Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.

The **value of the fixed rate cash flows** requires the discounting of each coupon and the final payment:

$$B_{Fixed} = \sum_{i=1}^{n} ke^{-r_i t_i} + Le^{-r_n t_n}$$

A floating-rate bond is worth the notional principal immediately after a payment because at this time the bond is a "fair deal" where the borrower pays LIBOR for each subsequent accrual period: $B_{Float} = L$. It follows that immediately before the payment, the value of the floating rate bond is the notional principal plus the floating payment $k^*$ that will be made at time $t_i$ (which was determined at the last payment date): $B_{Float} = L + k^*$

So, **valuing the floating-rate stream** is easier! We only need to discount the sum of the notional principal ($L$) and the next floating-rate payment ($k^*$):

$$B_{Float} = (L + k^*)e^{-r_1 t_1}$$

BIONIC TURTLE

# Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions (continued)

## Hull Example 7.2:

On a notional principal of $100 million, a swap is arrangement is made so as to receive 6-month LIBOR and pay 3% per annum (with semiannual compounding).

The swap has a remaining life of 1.25 years.

The LIBOR rates with continuous compounding for 3-month, 9-month, and 15- month maturities are 2.8%, 3.2%, and 3.4%, respectively. The 6-month LIBOR rate at the last payment date was 2.9% (with semiannual compounding).

Hull Ex. 7.2: Valuing a swap in terms of bonds ($ millions).

| Assumptions | |
|---|---|
| Notional | $100.00 |
| Swap rate | 3.00% |

| Time | LIBOR |
|---|---|
| 0.25 | 2.80% |
| 0.50 | 2.90% |
| 0.75 | 3.20% |
| 1.25 | 3.40% |

| Time | 0.25 | 0.75 | 1.25 |
|---|---|---|---|
| LIBOR | 2.80% | 3.20% | 3.40% |
| Discount Factor (CC) | 0.9930 | 0.9763 | 0.9584 |

| IRS value as two bonds (Hull Example 7.2) | | | | |
|---|---|---|---|---|
| **Floating Cash Flows** | | | | |
| Future value (FV) | $101.45 | | | |
| Present value (PV) | $100.74 | | | $100.74 |
| **Fixed Cash Flows** | | | | |
| Future value (FV) | $1.50 | $1.50 | $101.50 | |
| Present value (PV) | $1.49 | $1.46 | $97.28 | $100.23 |
| | | | | $0.5117 |

# Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions (continued)

| Assumptions | |
|---|---|
| Notional | $100.00 |
| Swap rate | 3.00% |

| Time | -0.25 | 0.25 | 0.75 | 1.25 | |
|---|---|---|---|---|---|
| LIBOR | 2.90% | 2.80% | 3.20% | 3.40% | |
| Discount Factor (CC) | | 0.9930 | 0.9763 | 0.9584 | |
| **IRS value as two bonds (Hull Example 7.2)** | | | | | |
| **Floating Cash Flows** | | | | | |
| Future value (FV) | | $101.45 | | | |
| Present value (PV) | | $100.74 | | | **$100.74** |
| **Fixed Cash Flows** | | | | | |
| Future value (FV) | | $1.50 | $1.50 | $101.50 | |
| Present value (PV) | | $1.49 | $1.46 | $97.28 | **$100.23** |
| | | | | | **$0.5117** |

# Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions (continued)

**Skipped in video (in slides for backup only)**

- The swap rate is 3% per annum and hence the semiannual coupon payments are \$1.5(=100 x 3%/2). So, the fixed-rate bond will have cash flows of 1.5, 1.5, and 101.5 in 3, 9 and 15 months respectively. The discount factors for these cash flows are calculated (for eg. the 3-month factor is $e^{-2.8\%*0.25}$ =0.9930) and multiplied with their respective cash flows. This gives us **present value of the (received or incoming) fixed cash flow stream** which is \$100.23 million.

- For the **present value of the floating-rate cash flow stream,** we only need to value one cash flow at three months: the next floating payment of \$1.45 ($k^*$) to be made in 6 months, (because it's a semi-annual payment on 2.9%, it is the rate of 1.45% times the notional) plus the notional ($L$=\$100) which equals \$101.45. That's the future value in three months, so we discount it with the 3-month factor (0.9930) to get its present value of \$100.74 million.

- The **value of the swap**, to our fixed rate payer, is the difference between the present value of the floating rate stream they are receiving (100.74) and the present value of the fixed cash flow stream they are paying (100.23), which is \$0.5117 million.

# Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).

## Valuation as a sequence of forward rate agreements (FRAs)

**Hull Ex. 7.3: Valuing a swap in terms of FRAs ($ millions)**

| Assumptions | |
|---|---|
| Notional | $100.00 |
| Swap rate | 3.00% |

| Time | LIBOR |
|---|---|
| 0.25 | 2.80% |
| 0.50 | 2.90% |
| 0.75 | 3.20% |
| 1.25 | 3.40% |

| Time | 0.25 | 0.75 | 1.25 |
|---|---|---|---|
| LIBOR | 2.80% | 3.20% | 3.40% |
| Discount Factor (CC) | 0.9930 | 0.9763 | 0.9584 |

**IRS value as FRAs (Hull Example 7.3)**

| Time | 0.25 | 0.75 | 1.25 | |
|---|---|---|---|---|
| LIBOR (continuous) | 2.80% | 3.20% | 3.40% | |
| Forward rates (CC) | 2.80% | 3.40% | 3.70% | |
| Forward rates (s.a.) | 2.90% | 3.43% | 3.73% | |
| Floating CFs (FV) | $1.45 | $1.71 | $1.87 | |
| Fixed CFs (FV) | $1.50 | $1.50 | $1.50 | |
| Net cash flows (FV) | -$0.05 | $0.21 | $0.37 | |
| Net cash flows (PV) | -$0.05 | $0.21 | $0.35 | $0.5117 |

- Use the LIBOR/swap zero curve to calculate forward rates for each of the LIBOR rates that will determine swap cash flows.
- Calculate swap cash flows by assuming LIBOR rates will equal the forward rates.
- Discount the net cash flows (at LIBOR/swap zero rates) to obtain the swap value.

# Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs) (continued)

The key difference here is that we calculate the forward rate in 3 months and in 9 months; specifically, we want to calculate the 3 x 9 (6-month rate when contract expires in three months) and the 9 x 15 (6-month rate when contract expires in 9 months).

For e.g. the 3 x 9 forward rate in continuous terms is given by:

$$\frac{\boldsymbol{R_2 T_2 - R_1 T_1}}{\boldsymbol{T_2 - T_1}} = \frac{\mathbf{3.2\% \times 0.75 - 2.8\% \times 0.5}}{\mathbf{0.75 - 0.25}} = \mathbf{3.4\ \%}$$

This is converted to a semiannual basis as:

$$2 \times e^{3.4\%/2} - 1 = 3.43\%$$

Similarly, the 9 x 15 forward rate on a semiannual basis is calculated as 3.73%.

BIONIC TURTLE

## Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).

**Assumptions**

| | |
|---|---|
| Notional | **$100.00** |
| Swap rate | **3.00%** |

**IRS value as FRAs (Hull Example 7.3)**

| **Time** | **0.25** | **0.75** | **1.25** | |
|---|---|---|---|---|
| LIBOR (continuous) | 2.80% | 3.20% | 3.40% | |
| Forward rates (CC) | 2.80% | 3.40% | 3.70% | |
| Forward rates (s.a.) | 2.90% | 3.43% | 3.73% | |
| **Floating CFs (FV)** | $1.45 | $1.71 | $1.87 | |
| **Fixed CFs (FV)** | $1.50 | $1.50 | $1.50 | |
| Net cash flows (FV) | -$0.05 | $0.21 | $0.37 | |
| Net cash flows (PV) | -$0.05 | $0.21 | $0.35 | **$0.5117** |

# Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).

## Valuation as a sequence of forward rate agreements (FRAs)

**Hull 10th Edition, EOC Problem 7.2**

| **Assumptions** | |
|---|---|
| Notional | **$100.00** |
| Swap rate | **4.00%** |

| **Time** | **-0.167** | **0.333** | **0.833** |
|---|---|---|---|
| **6 mo FWD LIBOR (sa)** | **2.40%** | **3.00%** | **3.00%** |
| **OIS (cc)** | **2.70%** | **2.70%** | **2.70%** |
| **Discount Factor (CC)** | | 0.9910 | 0.9778 |

**Valuation as FRAs necessary here because LIBOR is not the discount rate**

**IRS value as FRAs (Hull Example 7.3)**

| **Time** | **0.333** | **0.833** | |
|---|---|---|---|
| Forward rates **(s.a.)** | 2.40% | 3.00% | |
| **Floating CFs (FV)** | $1.20 | $1.50 | |
| **Fixed CFs (FV)** | $2.00 | $2.00 | |
| Net cash flows (FV) | -$0.80 | -$0.50 | |
| Net cash flows (PV) | -$0.7928 | -$0.4889 | **-$1.2817** |

# Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs) (continued)

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From the table, the cash flows exchanged in 3 months are:

- The **fixed rate** of 3% (semiannual rate of 1.5%) will lead to a cash outflow of =$1.5 million (=100 x 3%/2)
- The **floating rate** of 2.9% (that set 3 months ago) will lead to a cash inflow of $1.45 million (=100 x 2.9%/2).

The cash flows exchanged in 9 months assuming that forward rates are realized are

- The **cash outflow** is $1.5 million as before.
- The **cash inflow** using 3 x 9 forward rate of 3.43% is 1.71 million (=100 x 3.43%/2)

BIONIC TURTLE

# Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs) (continued)

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- Similarly, the cash flows that will be exchanged in 15 months assuming that forward rates are realized are: cash outflow of $1.5 million and a cash inflow of $1.87 million.

- The differences between the inflows and outflows of the 3, 9 and 15-month cash flows gives us the net cash flows. These when discounted by their discount factors (0.9930, 0.9763 and 0.9584, respectively) gives us the present value of the net cash flows which is equal to $0.5117 million.
- This value of the swap calculated as a series of FRA's is in agreement with the value calculated in the previous section by decomposing the swap into bonds.

# Explain the mechanics of a currency swap and compute its cash flows.

**A currency swap exchanges principal and interest in one currency for principal and interest in another currency**.

- A currency swap agreement requires the principal to be specified in each of the two currencies. The principal amounts are exchanged at the beginning and at the end of the life of the swap.
- For a party paying interest in the foreign currency, the foreign principal is received, and the domestic principal is paid at the beginning of the swap's life. At the end of the swap's life, the foreign principal is paid and the domestic principal is received.
- Usually the principal amounts are chosen to be approximately equivalent using the exchange rate at the swap's initiation. When they are exchanged at the end of the life of the swap, their values may be quite different.

BIONIC TURTLE

# Explain the mechanics of a currency swap and compute its cash flows (continued)

**Currency Swaps**

Each of the three types of currency swaps below would involve an initial **exchange of principal** in the opposite direction to the interest payments and a **final exchange of principal** in the same direction as the interest payments at the end of the swap's life.

- **Fixed-for-fixed currency swap:** exchanging principal and interest payments at a fixed rate in one currency for principal and interest payments at a fixed rate in another currency.
- **Fixed-for-floating currency swap:** floating interest rate in one currency is exchanged for a fixed interest rate in another currency.
    - ❑ Can be regarded as a portfolio consisting of a fixed-for-fixed currency swap and a fixed-for-floating interest rate swap.
- **Floating-for-floating currency swap:** a floating interest rate in one currency is exchanged for a floating interest rate in another currency.
    - ❑ Can be regarded as a portfolio consisting of a fixed-for-fixed currency swap and two interest rate swaps, one in each currency.

# Explain the mechanics of a currency swap and compute its cash flows (continued)

**Example:**

Consider a 5-year currency swap with a company:

- Paying a fixed rate of interest of 5.0% in sterling, and
- Receiving a fixed rate of interest of 6.0% in dollars from its counterparty.
- Interest rate payments are made once a year and the principal amounts exchanged are $15 million and £10 million.

**Cash flows in a currency swap**

| | | |
|---|---|---|
| | Receive dollars @ | 6.00% |
| | Pay sterling @ | 5.00% |
| **Period Year** | **Dollar Cash Flows** | **Sterling Cash Flows** |
| 0 | -$15.00 | £10.00 |
| 1 | $0.90 | -£0.50 |
| 2 | $0.90 | -£0.50 |
| 3 | $0.90 | -£0.50 |
| 4 | $0.90 | -£0.50 |
| 5 | $15.90 | -£10.50 |

This is a **fixed-for-fixed** currency swap because the interest rate in each currency is at a fixed rate.

- At the outset of the swap, our company pays $15 million and receives £10 million. Each year during the life of the swap contract, it receives $0.90 million (=6% x $15) and pays £0.50 million (=5% x £10). At the end of the life of the swap, it pays a principal of £10 million and receives a principal of $15 million.

BIONIC TURTLE

# Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows.

**A currency swap can be used to transform borrowings (liability) in one currency to borrowings in another.**

- Continuing with the previous example, a company issues $15 million of US- dollar-denominated bonds at 6% interest.

  The swap has the effect of transforming this transaction into one where it has borrowed £10 million at 5% interest. The initial exchange of principal converts the proceeds of the bond issue from US dollars to sterling. The subsequent exchanges in the swap have the effect of swapping the interest and principal payments from dollars to sterling. The resulting cash flows are as shown in the previous section.

**The swap can also be used to transform the nature of assets.**

- Suppose if the same company can invest £10 million in the UK to yield 5% per annum for the next 5 years, but feels that the US dollar will strengthen against sterling and prefers a US-dollar-denominated investment. The swap has the effect of transforming the UK investment into a $15 million investment in the US yielding 6%.

BIONIC TURTLE

## Calculate the value of a currency swap based on two simultaneous bond positions.

**A currency swap exchanges principal and interest in one currency for principal and interest in another currency.** The valuation of currency swap is given by:

$$V_{SWAP} = B_D - S_0 B_F$$
$$V_{SWAP} = S_0 B_F - B_D$$

- In the first case, the valuation of a swap that pays in foreign currency and receives US dollars involves subtracting the foreign bond after translating its value based on the spot exchange rate (expressed as number of dollars per unit of foreign currency). In effect, the exchange rate allows you to "standardize" on US dollars and take the difference in values.
- The second one shows the value of a swap where the foreign currency is received and dollars are paid. The value of a swap can therefore be determined from interest rates in the two currencies and the spot exchange rate.

BIONIC TURTLE

# Calculate the value of a currency swap based on two simultaneous bond positions (continued)

**Hull Example 7.4:**

**Hull Example 7.4:** Consider a company that has entered into a currency swap in which it receives 5% per annum in yen and pays 8% per annum in dollars once a year. The principals in the two currencies are $10 million and 1,200 million yen.

The term structure of interest rates is flat in both Japan and the United States at 4% and 9% per annum respectively (both with continuous compounding). The swap will last for another 3 years, and the current exchange rate is 110 yen = $1.

| Assumptions | US | Yen |
|---|---|---|
| Principal | $10.00 | ¥1,200.0 |
| Swap (fixed) rates | 8.0% | 5.0% |
| Interest rate | 9.0% | 4.0% |
| FX exchange rate, USDJPY | $0.00909 | ¥110.0 |
| | *JPYUSD* | *USDJPY* |

| Time | 1.0 | 2.0 | 3.0 | 3.0 |
|---|---|---|---|---|
| **US Rate** | 9.0% | 9.0% | 9.0% | 9.0% |
| **Japanese Rate** | 4.0% | 4.0% | 4.0% | 4.0% |

BIONIC TURTLE

# Calculate the value of a currency swap based on two simultaneous bond positions (continued)

**Hull Example 7.4:**

| Time | 1.0 | 2.0 | 3.0 | 3.0 | |
|---|---|---|---|---|---|
| US Rate | 9.0% | 9.0% | 9.0% | 9.0% | |
| Japanese Rate | 4.0% | 4.0% | 4.0% | 4.0% | |

| Value currency swap as two bonds | | | | | |
|---|---|---|---|---|---|
| **Pay Dollars** | | | | | |
| Future value (FV) | $0.80 | $0.80 | $0.80 | $10.00 | |
| Present value (PV) | $0.73 | $0.67 | $0.61 | $7.63 | **$9.64** |
| **Pay Yen** | | | | | |
| Future value (FV) | ¥60.00 | ¥60.00 | ¥60.00 | ¥1,200.00 | |
| Present value (PV) | ¥57.65 | ¥55.39 | ¥53.22 | ¥1,064.30 | **¥1,230.55** |
| **Present value (PV), US Dollars** | | | | → | **$11.19** |
| **Net Value** | | | | | **$1.543** |

| Value currency swap as FRAs | | | | | |
|---|---|---|---|---|---|
| **Pay Dollars** | | | | | |
| **Future value (FV), (a)** | **$0.80** | **$0.80** | **$0.80** | **$10.00** | |
| **Pay Yen** | | | | | |
| Future value (FV), Yen | ¥60.00 | ¥60.00 | ¥60.00 | ¥1,200.00 | |
| Forward Rate (IRP) | $0.009557 | $0.010047 | $0.010562 | $0.010562 | |
| **$ value of Yen CF (FV), (b)** | **$0.57** | **$0.60** | **$0.63** | **$12.67** | |
| **Net Cash Flow (FV) = (b-a)** | **-$0.23** | **-$0.20** | **-$0.17** | **$2.67** | |
| Net Cash Flow (PV) | **-$0.21** | **-$0.16** | **-$0.13** | **$2.04** | **$1.543** |

BIONIC TURTLE

# Calculate the value of a currency swap based on two simultaneous bond positions (continued)

**Hull Example 7.5:**

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The calculations are shown in the table below.

**Hull Ex 7.4: Valuation of currency swap in terms of bonds ($ millions)**

| | | | | | |
|---|---|---|---|---|---|
| **Pay Dollars** | | | | | |
| Future value (FV) | $0.80 | $0.80 | $0.80 | $10.00 | |
| Present value (PV) | $0.73 | $0.67 | $0.61 | $7.63 | **$9.64** |
| **Pay Yen** | | | | | |
| Future value (FV) | ¥60.00 | ¥60.00 | ¥60.00 | ¥1,200.00 | |
| Present value (PV) | ¥57.65 | ¥55.39 | ¥53.22 | ¥1,064.30 | **¥1,230.55** |
| **Present value (PV), US Dollars** | | | | → | **$11.19** |
| **Net Value** | | | | | **$1.543** |

- The cash flows from the dollar bond underlying the swap are $0.80 million (=$10 x 8%) each year. The dollar principal of $10 million is paid in year 3. The present value of these cash flows paid using the dollar discount rate of 9% sum up to $9.64 million.
- The cash flows from the yen bond underlying the swap are ¥60 million (=¥1200 x 5%) each year. The yen principal of ¥1200 million is received in year 3. The present value of the cash flows received using the yen discount rate of 4% sum up to ¥1230.55 million.

The value of the swap (that pays in dollars and receives in yen) in dollar terms is therefore calculated as:

$$V_{SWAP} = S_0 B_F - B_D = ¥1230.55 \;/\; ¥110 \; - \$9.64 \;\rightarrow \$11.19 - \$9.64 \; = \$1.543$$

# Calculate the value of a currency swap based on a sequence of FRAs (continued)

**Hull Example 7.5:**

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The cash flows paid are \$0.80 million each year plus the additional dollar principal of \$10.0 million paid in year 3. The cash flows received are ¥60.0 million each year plus the additional yen principal of ¥1200.0 in year 3. The current spot rate is 0.009091 dollar per yen. From $r$ = 9% and $r_f$ = 4%, the forward rates for 3 years are calculated.

- For eg. 1-year forward rate is calculated as: $F_0 = S_0 e^{(r - r_f)T} = 0.009091 \times e^{(9\% - 4\%) \times 1} = 0.009557$

**Assuming that these forward rates are realized, the swap can be valued.**

- For example, if the 1-year forward rate is realized, the yen cash flow in year 1 is worth: \$0.57 million (=60 x 0.009557).
- The net cash flow at the end of year 1 is \$0. 23 million ($b - a$ = 0.57 - 0.8). This has a present value of \$0.21 million ($0.23 \times e^{-9\% \times 1}$).
- The values of the other forward contracts are calculated similarly and summed up to obtain the total value of the swap which is \$1.5430 million. Thus, the total value of the swap calculated as a series of currency forward contracts agrees with the value of the swap calculated by decomposing it into bonds.

BIONIC TURTLE

# Describe the credit risk exposure in a swap position.

**Because a swap involves offsetting position, there is no credit risk when the swap has negative value.** A financial institution is liable to lose money if one of the counterparties defaults when it has positive value in its swap because it still has to honor its swap agreement with the other counterparty.

- For an ***interest rate swap***, because principal is not exchanged at the end of the life of an interest rate swap, the potential default losses are much less than those on an equivalent loan.
- In a ***currency swap***, the risk is greater because currencies are exchanged at the end of the swap.
  - ❑ the payer of the currency with the ***high interest rate*** is in the position where the forward contracts corresponding to the early exchanges of cash flows have negative values, and the forward contract corresponding to final exchange of principals has a positive value.
  - ❑ the payer of the currency with the ***low interest rate*** is in the opposite position where the forward contracts corresponding to the early exchanges of cash flows have positive values, while that corresponding to the final exchange has a negative value.

BIONIC TURTLE

# Identify and describe other types of swaps, including commodity, volatility and exotic swaps.

## Other Types of Swaps

**Variations of standard fixed for floating interest rate (or plain vanilla) swaps:**

- Instead of the common floating reference rate LIBOR, commercial paper rates may be used
- tenor (payment frequency) can vary from the normal 6 month LIBOR to 1 month, 3 months, and 12 month LIBOR
- the tenor on the floating side does not have to match the tenor on fixed side.

Principal can vary throughout the swap term:

- **Amortizing Swap:** Principal reduces in a predetermined way to correspond to the amortization schedule on a loan
- **Step-up swap:** Principal increases in a predetermined way
- **Deferred (forward) swap:** the parties do not begin exchange until some future period. Sometimes the principal to which the fixed payments are applied is different from the principal to which the floating payments are applied.

BIONIC TURTLE

# Identify and describe other types of swaps, including commodity, volatility and exotic swaps (continued)

**Constant maturity swap (CMS swap):** an agreement to exchange a LIBOR rate for a swap rate.

**Compounding swap**: Interest on one or both sides is compounded forward to the end of the swap's life.

**Accrual swap:** Interest on one side of a swap accrues only if floating rate is within a certain range.

**Constant maturity Treasury swap (CMT swap):** the counterparties agree to swap a LIBOR rate for a Treasury rate.

**Diff swap or Quanto:** A rate observed in one currency is applied to a principal amount in another currency.

**Equity swaps:** Agreement to exchange total return realized on an equity index in exchange for LIBOR.

**Commodity swaps:** A series of forward contracts on a commodity with different maturity dates and the same delivery prices.

BIONIC TURTLE

# Identify and describe other types of swaps, including commodity, volatility and exotic swaps (continued)

**Exotic swaps:** Swaps are limited only by the imagination of financial engineers and the desire of corporate treasurers and fund managers for exotic structures. Eg. A 5/30 swap entered into between Procter and Gamble and Bankers Trust, where payments depend in a complex way on the 30−day commercial paper rate, a 30−year Treasury bond price, and the yield on a 5−year Treasury bond.

**LIBOR−in arrears swap:** The LIBOR rate observed on a payment date is used to calculate the payment on that date

**Options embedded in swaps like**

- Extendable swap − one counterparty can choose to extend the life of the swap
- Puttable − one party has the option to terminate early
- Swaptions − options on swaps.

**Volatility swaps:** In a series of time periods, at the end of each period, one side pays a preagreed volatility, while the other side pays the historical volatility realized during the period. Both volatilities are multiplied by the same notional principal in calculating payments.

## The End

**P1.T3. Financial Markets & Products**

Hull, Options, Futures & Other Derivatives, 9th Edition

Swaps